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Numerical study on rectangular-convex-triangular profiles with all variable thermal properties

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A R T I C L E I N F O

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A B S T R A C T

We present solutions of the nonlinear mathematical models for rectangular, convex and triangular fins with variable thermal conductivity, heat transfer coefficient, surface emissivity and heat generation. The model for rectangular fins is a nonlinear and non-singular value energy equation. The convex and triangular fins are described by a nonlinear and singular value energy equation. The boiling regimes, such as film boiling, nucleate boiling and laminar natural convection are governed by the value of the heat transfer coefficient power index. The non-singular and singular energy balance equation corresponding to rectangular and non-rectangular profiles are solved separately using the classical Adomian decomposition method and the modified Adomian decomposition method respectively. The results are validated by solving the equations independently using the spectral quasi-linearization method. The effects of various thermo physical parameters such as the thermal conductivity parameter, environmental temperature and heat generation parameter on the temperature distribution is analyzed.

1. Introduction

Fins are extended surfaces used to dissipate heat from the primary surface to the surrounding fluid. The study of various fin profiles, such as triangular, convex, etc. is a topic of current research interest. The efficient material utilization and their lightweight are paramount factors, particularly for airborne and space applications. There is no doubt that the rectangular fin is simple in shape and, inspite of its higher relative weight as compared to parabolic or triangular fins, the results for rectangular fins are necessary for comparison and completeness. Kem and Kraus [1] presented an extensive review related to this topic.

In many engineering applications where the weight of a fin structure is the decisive parameter, fins of concave profiles are the lightest among the profiles. However, due to complex curvature, the fin with a concave parabolic profile is difficult to manufacture and therefore more expensive. The convex parabolic profile and straight triangular fin are a better alternative to concave profiles. Most studies on the thermal performance of fins take consider fins of uniform cross-sectional area, uniform temperature distribution and constant thermo physical properties in order to reduce the complexity of the mathematical equations. The variation in the cross-sectional area introduces a singularity in the governing equation. If the singular value problem does not contain any nonlinear terms, the solution is obtained in terms of standard mathematical functions such as Bessel functions and hyperbolic functions. The assumption of uniform temperature distribution and constant thermo physical properties is inconsistent with the thermal performance of fins under realistic operating conditions; this invariably involves nonlinearities and singularities. There are instances when large temperature difference exists in the fin, where the thermal conductivity of the fin is a function of the temperature. For instance, the thermal conductivity of copper decreases from 482 W/m °C at 100 K to 366 W/m °C at 800 K. On the other hand, the thermal conductivity of AISI-302stainless steel increases from 17.3 W/m °C at 400 K to 25.5 W/m °C at 1000 K [2–3]. Arslanturk [4] used the classical Adomian decomposition method (ADM) to evaluate the efficiency of a conductive–convective straight fin with variable thermal conductivity. Moitsheki [5] studied the longitudinal fins with triangular and parabolic profiles and variable thermal conductivity and heat transfer coefficient.Mallick et al. [6] applied the classical Adomian decomposition method (ADM) to evaluate the thermal stresses in an annular fin with temperature dependent thermal conductivity. Kundu and Wongwises [7] employed the Adomian decomposition method to evaluate the thermal performance of a longitudinal plate with variable thermal conductivity and heat transfer coefficient.

The modes of heat dissipation from the fin surface include convection, radiation and condensation boiling or combinations of these. For example, the values of the exponent, n are −0.25, 0.25, 0.33, 2 and 3 for condensation, laminar natural convection, turbulent natural convection,

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Nomenclature

N_r	Radiation-conduction parameter
Nc	Convective-conduction parameter
C	Constant which represents the temperature
k	Temperature dependent thermal conductivity, $W/(mK)$
k_a	Thermal conductivity corresponding to ambient condition $T_a, W/(mK)$
ε_s	The surface emissivity corresponding to radiation sinks temperature, T_s
T	Temperature, K
P_x	Fin perimeter of element at location x, m
T_b	Fin's base temperature, K
T_a	Convection sink temperature, K
T_s	Radiation sink temperature, K
L	Length of the fin, m
x	Axial variable, m
A_x	Cross-sectional-area of the element at location x, m^2
X	Dimensionless axial co-ordinate
A	Thermal conductivity parameters
B	The surface emissivity parameters
N_r	Radiation-conduction parameter
G	Heat generation number
ε_G	Heat generation parameter

Greek symbols

α	Slope of the thermal conductivity-temperature curve, K^{-1}
β	Slope of the surface emissivity-temperature curve, K^{-1}
γ	Slope of the heat generation-temperature curve, K^{-1}
θ	Dimensionless temperature of the fin,
θ_a	Dimensionless convection sink temperature
θ_s	Dimensionless radiation sinks temperature,
σ	Stefan-Boltzmann constant
ε	Emissivity

nucleate boiling and radiation respectively. Bhowmik et al. [8] studied the performance of rectangular and hyperbolic fin profiles with variable thermal conductivity and heat transfer coefficient. They employed the classical Adomian decomposition method (ADM) to solve the rectangular profile problem, and the double decomposition method (DDM) for hyperbolic profiles. Darvishi et al [9] presented a numerical solution of the hyperbolic annular fin with temperature dependent thermal conductivity. Singla and Das [10] presented a decomposition solution for a stepped fin with all temperature dependent mode of heat transfer. Mosayebidorcheh et al. [11] used the differential transform method (DTM) to solve the nonlinear heat transfer equation with power law variation in thermal conductivity and heat transfer coefficient.

When fins dissipate heat by natural convection, radiation heat loss must be taken into account in order to reduce the error in calculations. Aziz and Torabi [12] studied the convective-radiative rectangular fin problem with variable thermal conductivity, heat transfer coefficient and surface emissivity. Torabi et al. [13] presented comparative study of convective-radiative longitudinal fins of rectangular, trapezoidal and concave profiles assuming variable thermal properties. Similarly, an evaluation of the thermal performance and efficiency of rectangular, triangular, convex and exponential profiles with variable thermal properties was presented in [14]. Mosayebidoechehet al. [15] investigated the performance of convective-radiative longitudinal fins of various profiles made of different materials. Sun et al. [16] used the spectral collocation method (SCM) to study the temperature distribution of convective-radiative rectangular fins. Aziz and Bouaziz [17] used an optimal linearization method to study the performance and design criteria of rectangular fins with variable thermal conductivity and heat generation. Mallick et al. [18] predicted the heat generation number of a rectangular fin with variable thermal conductivity and temperature dependent internal heat generation using the homotopy perturbation method.

In most of the studies, it is assumed that the velocity of the stretching sheet varies linearly with the distance from the extrusion slit. Many practical situations involve nonlinear stretching sheet in industrial manufacturing processes, such as hot rolling, the cooling of metallic plates, wire drawing and glass fiber production, aerodynamic extrusion of plastic sheets. Pal and Mondal [19] studied the hydromagnetic non-Darcy flow and heat transfer over a stretching sheet with thermal radiation and Ohmic dissipation. Acharya et al. [20] analyzed the flow of a magnetized upper-convected Maxwell nanofluid flow over an inclined stretching sheet. Khan et al. [21-24] investigated the three-dimensional flow and heat transfer to burgers fluid and Sisko fluid flow using Cattaneo–Christov heat flux model.

The main objective of this study is to use the classical Adomian decomposition method to obtain near-closed form solutions of the rectangular fin problem, and the modified Adomian decomposition method in the case of convex and triangular fins with variable thermal properties. A secondary focus of the study is to use a recent spectral quasilinearization method (SQLM) in the study of fin problems [25,26]. A comparison of the results shows that the SQLM gives high accuracy and fast convergence with relatively few spatial grid points. Numerous contributions have been made in solving the fin problem with variable parameters using various semi-analytical approaches [27]. The modified decomposition method can be applied to any singular value problem, Adomian [28]. The method gives rapid convergence and has several advantages over other semi-analytical approaches.

2. Mathematical formulations

Consider longitudinal straight fins of length L and width b.A_x and perimeter P_x are the cross sectional area and perimeter of at a distance x from the free end of the fin. For mathematical convenience we locate the origin of the co-ordinate (x) at the fins tip. The base temperature of the fin is maintained at T_b. The fin transfer heat by convection to an environmental temperature T_a and by radiation to an effective sink temperature T_s. The fin has some internal heat source $q(T)$ which is function of temperature. The steady state one dimensional energy equation for any profile is with all variable thermal properties given by

$$\frac{d}{dx}\left[k(T)A_x\frac{dT}{dx}\right]-h(T)P_x\left(T-T_a\right)-\varepsilon(T)\sigma P_x\left(T^4-T_s{}^4\right)+q(T)A_x=0 \tag{1}$$

With the insulated boundary conditions

$$\frac{dT}{dx}=0 \quad at \quad x=0 \tag{2a}$$

$$T=T_b \quad at \quad x=L \tag{2b}$$

The thermal conductivity, heat transfer coefficient, surface emissivity and heat generation of any metal assumed to be temperature dependent.

$$k(T)=k_a\left[1+\alpha\left(T-T_a\right)\right] \tag{3a}$$

$$h(T)=h_b\left(\frac{T-T_a}{T_b-T_a}\right)^n \tag{3b}$$

$$\varepsilon(T)=\varepsilon_s\left[1+\beta\left(T-T_s\right)\right] \tag{3c}$$

$$q(T)=q_0\left[1+\gamma\left(T-T_a\right)\right] \tag{3d}$$

Rectangular, convex and triangular fin profile fin profiles are categorized in the variation of fin thickness along its extended length as shown in Fig. 1(a), (b) and (c) respectively. Mathematically, $A_x = bt(x)$ Where b is the width of the fin is same for all profiles, $t(x)$ is the fin thickness