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Extension of dual equivalent linearization to nonlinear analysis of thermal behavior of a two-node model for small satellites in Low Earth Orbit

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A B S T R A C T

The purpose of this study is to extend the dual equivalent linearization to the problem of thermal analysis of small satellites in Low Earth Orbit (LEO) using two-node model. The satellite is modeled as a body with two iso-thermal nodes, namely, outer and inner ones. The outer node represents for the shell of the satellite whereas the inner node stands for some inner equipment. In space environment, the outer node is subjected to three main thermal loadings including the solar irradiation, albedo and Earth's infrared radiation. Also, the inner node is assumed to be undergone a constant heat dissipation level. To simplify the process of linearization, a preprocessing step in separating nonlinear terms of the original system is carried out to get an equivalent system in which each differential equation contains only one nonlinear term. Based on the dual criterion a closed form of linearization coefficients system is obtained and solved by a Newton–Raphson iteration procedure. It is shown that the solutions obtained from the dual criterion are in a good agreement with those obtained from the Grande's approach and Runge–Kutta algorithm.

1. Introduction

In space engineering, the operation of any kind of equipment will be failed if subjected to an extreme temperature environment condition in a long time. For satellite equipment, to ensure that they operate exactly in appropriate temperature ranges, one needs to control their thermal characteristics using various thermal techniques. Therefore, the analysis of thermal problem is one of the most important tasks in processes of designing, manufacturing and launching a satellite into its orbit [1,2]. Due to the complexity of thermal models of real satellites, to find their thermal behavior, one can use available commercial software tools [1,3–5]. The numerical solvers integrated in the softwares can provide solutions of the problem, such as evolutions of nodal temperatures in time, in a convenient manner. To understand some important properties of satellite thermal, however, many analytical studies on specific thermal models are necessary for the primary stage of thermal design when the concept of the satellite is opened. In [6], Oshima and Oshima presented an analytical approach for thermal design of spacecraft in several special cases of thermal balance equations. Recently, several techniques in analyzing satellite thermal have been proposed, for example, the techniques in [7,8]. Arduini et al. suggested a technique of linearization [7] for solving the inverse problem in the satellite thermal control. Grande et al. [8] studied a technique of linearization for a problem of thermal mathematical model for small satellites in the Low Earth Orbit (LEO). They used Fourier's expansions for space environment thermal loadings in order to obtain a vibrating system similar to the one in conventional solid mechanics with forced excitations. In [9,10], Gaite et al. also solved the problem of Grande [8] using several other analytical tools, such as perturbation and averaging methods. Their results are then generalized to models of many-node [11].

More recently, Anh et al. [12] have developed an analytical technique for analyzing thermal radiation characteristics of small satellites with single-node model based on the equivalent linearization concept in random vibration. They use the dual criterion of linearization to find approximate solutions of a nonlinear differential equation corresponding to the single-node model. They proposed a compact dual criterion which is a combination of both conventional and dual versions of equivalent replacement. The term "conventional linearization" addresses the linearization approach based on the mean-square criterion in which the error between the original nonlinear system and its linearized equation is minimum whereas the external loadings of both systems are the same [13–16]. Also, the "dual criterion" is based on two steps of equivalent replacement. The first step is carried out so that the nonlinear function of the original system is replaced by a linear one. In the second step, the obtained linear function is replaced by another nonlinear one that

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belongs to the same class of the original nonlinear function [17,18]. The application potential of the equivalent linearization method is of promise. For the field of satellite thermal analysis, finding an appropriate linearization method giving high precision of the solution is still an open problem because of strong nonlinearity of the thermal system in which traditional approaches may lead to large errors during the linearization treatment. For more information about the development of equivalent linearization method, readers can see in the book of Roberts and Spanos [19], and papers published recently [20–26].

Naturally, the dual approach for the problem of small satellites' thermal behavior with single-node model [12] can be extended to the many-node model. For that purpose, the present study is devoted to an extension of the dual criterion to find approximate solutions of a two-node model of satellite thermal in LEO. In practice, for systems with high dimensions (larger or equal to three), analytical approaches may lead to difficulty and complicatedness in calculations. Therefore, the single-node and two-node models appear suitable in analytical investigations. Our obtained results for two-node model of satellite thermal using the dual criterion show the high accuracy in comparison with other approaches such as the conventional linearization and Grande's approach.

The two-node thermal model in this study is suitable for small compact spinning satellites. The term "small satellite" refers to satellites with the following subgroups: nano- and pico-satellite (< 10 kg), micro-satellite (10–100 kg), and mini-satellite (100–500 kg) [2,27,28]. The medium and large satellites have mass values larger than 500 kg and 1000 kg, respectively. Over last decades, the number of small satellites has considerably increased in different space missions. This can be explained from aspects of moderate enough cost and short development time of small satellites, especially for nano- and pico-satellites developed by research groups at universities [29]. The term "spinning" indicates the rotation motion of satellite around one of its axes with a defined angular velocity when traveling in its orbit. The researches on thermal distribution of spinning spacecrafts can be seen in [5,30–32]. As the spinning axis of satellite is remained perpendicular to solar rays during motion, a featured characteristic of spinning satellite is that the thermal distribution on its outer shell is nearly uniform because of the temperature homogenization effect when rotating with a large enough value of spinning speed [5]. This effect is used in the passive thermal control method for small satellites. Using the thermal distribution effect of compact spinning satellites, Grande et al. [8] have proposed a simplified model of satellite with two thermal nodes: the inner one includes all equipment within it, for example, batteries, power supply control unit, reaction wheels, on board computer; and the outer one represents for the shell, solar panels, and any external devices located on the outer surface of satellite. Our study on the thermal behavior of small satellites is based on that simplified model of Grande et al. For small compact spinning satellites, because the range of temperature variation (from minimum to maximum temperature) is narrow and contained in the designed temperature limits, the thermal properties of satellite's materials are assumed to be constant. Furthermore, the temperature gradients of outer shell can be neglected because of temperature homogenization effect of spinning satellites.

2. Two-node thermal balance model for small compact satellites in LEO

In this section, we consider a simple model of a small compact spinning satellite with two thermal nodes, namely, the outer and inner nodes. The geometric model of the satellite corresponding to the thermal mathematical model is illustrated in Fig. 1. Let C_1 and C_2 be thermal capacities of outer and inner nodes, respectively. The equation of the energy balance for the two-node model takes the following form [8]

$$\begin{aligned} C_1\dot{T}_1 &= k_{21}(T_2 - T_1) + r_{21}(T_2^4 - T_1^4) - A_{sc}\varepsilon\sigma T_1^4 + Q_s f_s(\nu t) \\ &\quad + Q_a f_a(\nu t) + Q_p, \\ C_2\dot{T}_2 &= -k_{21}(T_2 - T_1) - r_{21}(T_2^4 - T_1^4) + Q_{d2}, \end{aligned} \tag{1}$$

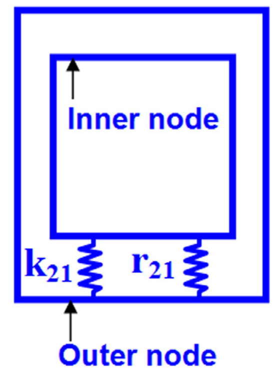

Fig. 1. A two-node thermal mathematical model.

(1)

where $T_1 = T_1(t), T_2 = T_2(t)$ are temperatures of outer and inner nodes, respectively. The quantities k_{21}, r_{21} are conductive and radiative coupling coefficients between two nodes. The notations ε, A_{sc} are the outer node's emissivity and radiative surface area values; $\sigma = 5.67 \times 10^{-8}$ $\mathrm{Wm^{-2}K^{-4}}$ is the Stefan–Boltzmann constant; Q_{d2} is the internal heat dissipation of inner node. The constant quantity Q_s is determined by the following expression

$$Q_s = G_s A_{sp} \alpha_s, \tag{2}$$

where G_s is the mean solar irradiation; A_{sp} is the satellite surface projected in the Sun's direction. For small compact spinning satellites in LEO, one can assume this area to be constant. The coefficient α_s is the solar absorptivity of the satellite surface area. Values of ε, α_s are taken in the interval [0, 1] and they depend on the property of surface materials of the satellite [2]. The solar irradiation thermal loading $Q_s f_s(\nu t)$ differs from zero and remains constant during illumination period P_{il}. It will vanish as the satellite is in the fraction of orbit in eclipse [9,10]. The mathematical representation for $f_s(\nu t)$ in an orbital period P_{orb} is as follows

$$f_s(\nu t) = \begin{cases} 1 & \text{if } \; 0 \le \nu t \le \mu\pi \;\text{ and }\; \left(1 - \frac{1}{2}\mu\right)2\pi \le \nu t \le 2\pi, \\ 0 & \text{if } \; \mu\pi < \nu t < \left(1 - \frac{1}{2}\mu\right)2\pi, \end{cases} \tag{3}$$

where $\nu = 2\pi/P_{orb}$; $\mu = P_{il}/P_{orb}$ is the ratio of the illumination period to orbital period. The quantity Q_a in Eq. (1) is determined by

$$Q_a = aG_s A_{sc} F_{scp} \alpha_s, \tag{4}$$

in which the albedo factor of the Earth, a, is estimated from the experimental data. Its value may be taken from 0.31 to 0.39 (see [2]). F_{scp} is the view factor from the whole satellite to the Earth. The albedo thermal loading depends on the orbit altitude and the solar zenith angle at satellite position [8]. The mathematical form of $f_a(\nu t)$ for an orbital period is as follows

$$f_a(\nu t) = \begin{cases} \cos(\nu t) & \text{if } \; 0 \le \nu t \le \frac{\pi}{2} \;\text{ and }\; \frac{3\pi}{2} \le \nu t \le 2\pi, \\ 0 & \text{if } \; \frac{\pi}{2} < \nu t < \frac{3\pi}{2}. \end{cases} \tag{5}$$

The Earth's infrared radiation Q_p can be evaluated as

$$Q_p = \varepsilon A_{sc} F_{scp} \sigma T_p^4, \tag{6}$$

where T_p is the Earth's equivalent black-body temperature. In next section, we will look for solutions of the system (1) using linearization techniques of approximate analysis.