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Surface energy effect on buckling behavior of the functionally graded nano-shell covered with piezoelectric nano-layers under torque

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ABSTRACT

In this paper, based on the electro-elastic surface/interface theory, the size-dependent effect on the torsional buckling behavior of functionally graded (FG) cylindrical nano-shell covered with piezoelectric nano-layers is studied. According to the classical shell theory together with von-Karman-Donnell type kinematics of nonlinearity, the primary formulations are given. The total energy of the nano-shell is derived by introducing the constitute relations for piezoelectric surfaces and interfaces. The principle of minimum potential energy is employed to establish the governing differential equations. An analytical solution is firstly presented, and then the generalized differential quadrature (GDQ) method is used to obtain the numerical results of nano-shells with different boundary conditions. Afterwards, the results without surface/interface effect are compared with the datum in the open literatures, and some numerical examples are presented to investigate the effect of surface/interface parameters and power-law index on the critical buckling load of the nano-shell.

1. Introduction

With the development of material science, piezoelectric materials are well-known for capturing the surrounding energy to produce the piezoelectricity, which makes them have a great number of applications in the smart devices [1–4]. In recent years, as many structures tend to miniaturization, piezoelectric nano-structures, such as nano-beams, nano-plates and nano-shells, have attracted wide attention in the micro-nano-electromechanical systems (MEMS/NEMS) [5–8]. Different from the piezoelectric bulks, piezoelectric nano-structures have unique electrical, mechanical and physical properties, which leads to many potential applications [5,6].

Due to the fact that the nano-structure is a system at the nanometer scale, the nano-sized effect cannot be ignored when predicting its behaviors, which has been demonstrated by experimental results [9,11] and atomistic simulations [10,12]. Since the classical theory is scale-free, it fails to predict the size-dependent response of nano-structures. Consequently, to consider the small scale effect, some non-classical theories are proposed, such as strain gradient theory [13–15], couple stress theory [16–18], nonlocal theory [19–21] and surface elasticity theory [22–24]. Among the non-classical theories, the surface elasticity theory raised by Gurtin and Murdoch has been widely used to study the vibration, bending, buckling and postbuckling behaviors of elastic nano-structures in recent years [25–38]. This can be attributed to the fact that in nano-structures, the ratio of atomic–size surface/interface to the volume is very high, and the behaviors of nano-structures are significantly related with the high surface/interface ratio. To describe the nano-sized effect, the surface/interface effect should be considered. Satisfactorily, the surface elasticity theory can make it by considering the surface/interface energy. However, it is worth noting that the surface elasticity theory cannot predict the effect of surface/interface piezoelectricity. To consider the size-dependent response of piezoelectric nano-structures, the electro-elastic surface/interface theory was introduced by Huang and Yu [39], and this theory is employed to carry out the present work.

In recent years, on the basis of non-classical theories, many theoretical investigations have been carried out for the elastic nano-structures. For example, based on the nonlocal theory and four different beam theories, the bending, buckling and vibration behaviors of nano-beams were studied by Reddy, and the effect of nonlocal parameter on the maximum deflection, critical buckling load and natural frequency were discussed [40]. According to the surface elasticity theory, the effect of surface stress on free vibration behavior of nano-plates was investigated by Ansari and Sahmani, and some numerical examples were presented to quantitatively evaluate the influences of surface parameters on the natural frequencies of nano-plates [26]. Besides, within the framework of surface elasticity theory, the nonlinear free vibration and nonlinear postbuckling behaviors of nano-plates were studied by Wang and Wang [29,37]. The nonlinear buckling and postbuckling behaviors of shear deformable nano-shell under radial compressive load were studied by

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using the surface elasticity theory, and the effect of surface free energy on the critical buckling load and end-shortening was discussed [38].

With the rapid development of piezoelectric materials, a number of works have also been carried out in the presence of different piezoelectric nano-structures. For instance, based on the consistent couple stress theory and Timoshenko beam theory, the electro-mechanical bending and vibration behaviors of a piezoelectric nano-beam were studied by Beni, and the effect of different parameters on the maximum deflection and natural frequency was presented [41]. Within the framework of electro-elastic surface/interface theory, the scattering of compressive waves around a piezoelectric nano-particle in a polymer matrix was studied, and the surface/interface effect on the electric displacement and dynamic stress around the nano-sized particle was presented [42]. According to the modified couple stress theory combined with Euler-Bernoulli beam theory, the electro-mechanical buckling analysis of a FG nano-bridge was studied by Shojaeian and Beni, and the influence of power law index, scale parameter and distance between two settled and movable electrodes was presented [43]. In addition, the size-dependent buckling, free vibration and bending behaviors of FG piezoelectric nano-beams were investigated by using the consistent couple stress theory and Euler-Bernoulli beam theory [44]. Zhang et al. studied the surface effect on the anti-plane shear wave propagation in an infinite piezoelectric nano-plate based on electro-elastic surface/interface model [45]. A size-dependent model for the vibration and buckling analysis of piezoelectric nano-plates was developed by using electro-elastic surface/interface theory [46].

Due to the fact that the nano-shell is a basic element in numerous nano-structures, a detailed and further study on nano-shells should be carried out to obtain the desired material properties and design the perfect nano-shell. In recent years, there have been some publications on the vibration [47–55] and buckling [56–63] behaviors of nano-shells. For example, a size-dependent model for vibration analysis of a sandwich nano-shell with FG materials was studied based on the couple stress theory [53]. The thermo-electro-mechanical vibration of a piezoelectric nano-shell with different boundary conditions was investigated by using the nonlocal theory and Love's thin shell theory [54]. According to nonlocal shell model, Ansari et al. studied axial buckling analysis of a multi-walled carbon nanotube under thermal load [57]. Based on surface/interface elasticity theory, the postbuckling behavior of an imperfect and pressured FG nano-shell in the thermal field was studied by Sahmani and Aghdam, and the effects of power-law index, thermal environments and geometric imperfection on the end-shortening and maximum deflection were discussed [59]. The size-dependent buckling models for a FG piezoelectric nano-shell under lateral pressure and an anisotropic piezoelectric nano-shell under combined axial and lateral loads were presented by using the new modified couple stress theory, and the influences of material length scale parameter, thickness and applied voltage on critical buckling load were analyzed [61,62]. Within the framework of surface elasticity theory, the nonlinear buckling and postbuckling analysis of a piezoelectric nano-shell under combined compressive load and electrical load was studied, and the effect of surface parameters and electrical load on the postbuckling load-deflection curves and load-shortening curves was discussed [63].

It is worth noting that the buckling of nano-shells is an essential matter. Due to the fact that it is very difficulty to obtain the stability solutions for a nano-shell subjected to torsional load, the torsional buckling analysis of nano-shells is more attractive, which can be found in some studies [64–74]. For instance, the buckling analysis of a single walled carbon nanotube subjected to torque was conducted by using the nonlocal elasticity theory and molecular dynamic simulation [65]. According to a continuum elastic double-shell model, the buckling and postbuckling analysis on a double-walled carbon nanotube under torque was investigated by Yao and Han, and the postbuckling relationship between the torque and the angle of twist was given [70]. On the basis of nonlocal elasticity and piezoelectricity theory, the torsional buckling analysis of an embedded armchair double-walled nanotube under electric load and thermal load was studied, and the influence of nonlocal parameter, temperature change, piezoelectric and dielectric constants on critical buckling load was presented [72]. A size-dependent model for the buckling analysis of a FG nano-shell under torque was developed by using the modified couple stress theory [74]. Through detailed calculations, the effect of length scale parameter, power-law index and aspect ratio on critical buckling load was discussed. Though some investigations have been performed on the torsional buckling analysis of nano-shells, up to present time, the electro-elastic surface/interface energy effect on the torsional buckling of a FG nano-shell with piezoelectric layers has not been carried out.

In the present study, the buckling behavior of a FG cylindrical nano-shell embedded with piezoelectric layers subjected to torque is investigated. The electro-elastic coupling surface/interface model is employed to derive the total energy of the nano-shell, and the principle of minimum potential energy is used to obtain the governing differential equations. Finally, some numerical results are illustrated to show the effects of surface/interface properties, geometric parameters and power-law index on the critical buckling load of the nano-shell.

2. Mathematical formulation

A FG cylindrical nano-shell embedded with piezoelectric nano-layers is considered. The length is Lnm. The thickness of FG layer is $2h$nm, and that of piezoelectric layers is $t$nm. The radius of mid-surface is denoted as R, as shown in Fig. 1. A twisting moment T is exerted on the two ends of nano-shell. With the origin of coordinate system located on the middle surface of nano-shell, the coordinates of a typical point in the axial, circumferential and radius directions are described by x, θ, and z, respectively. E_F and υ_F represent Young modulus and Poisson ratio of FG cylindrical nano-shell. E_P, υ_P, e_{31P}, e_{32P} and ξ_{33P} are Young modulus, Poisson ratio, piezoelectric and dielectric constants of piezoelectric layers.

With the high ratio of surface/interface to volume, the surface/interface plays an important role in predicting the buckling behavior of nano-shell. According to the electro-elastic surface/interface model, the surface/interface layers are different from the bulk, and possess their own electro-mechanical properties. In Fig. 1, the surfaces at inner and outer piezoelectric layers are represented by $S1$ and $S2$, respectively. The interface between the inner piezoelectric layer and the FG nano-shell is expressed by $I1$, and the interface between the outer piezoelectric layer and the FG nano-shell is depicted by $I2$. The material properties of surface are Lamé's constants λ^{Sk}, μ^{Sk}, residual stress σ_0^{Sk} and piezoelectric constants e_{31P}^{Sk}, e_{32P}^{Sk}. Those of interface are Lamé's constants λ^{Ik}, μ^{Ik} and residual stress σ_0^{Ik}, where k is equal to 1 or 2.

In this paper, the FG nano-shell is made up of two different fundamental materials. It is assumed that Young's modulus E_F and Poisson ratio υ_F of FG nano-shell vary continuously through the thickness direction. Following the works in Refs. [75,76], E_F and υ_F can be written as

$$E_F = \frac{9\mu K}{\mu + 3K}, \tag{1}$$

$$\upsilon_F = \frac{3K - 2\mu}{2(\mu + 3K)}, \tag{2}$$

where μ and K are, respectively, the effective shear modulus and bulk modulus of the FG nano-shell, and they can be obtained from the following formulations [77]

$$\mu = \frac{\Delta_1}{\Delta_2}, \tag{3}$$

$$K = \frac{1}{\Delta_3} - \frac{4}{3}\mu, \tag{4}$$

in which Δ_1, Δ_2 and Δ_3 are presented as

$$\Delta_1 = \frac{K_B}{K_B + \frac{4}{3}\mu} + \frac{5\mu_T}{\mu - \mu_T} + 2, \tag{5a}$$