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Size-dependent behavior of viscoelastic nanoplates incorporating surface energy and microstructure effects

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ABSTRACT

In this paper, the simultaneous effects of the microstructure rotation and surface energy on the behavior of nanoplates are investigated in the framework of viscoelasticity. Firstly, the modified couple stress elasticity and Gurtin-Murdoch surface elasticity theories are reconsidered and harnessed to incorporate respectively the viscoelastic microstructure local rotation and viscoelastic surface energy effects into the classical viscoelastic plate theory. The couple stress tensor is obtained incorporating measures for the elastic and the viscous behaviors of the plate. Surface stress tensor is derived depending on the surface elastic and viscous parameters. Afterwards, a variational approach on the basis of D'Alembert's principle in conjunction with the Kirchhoff plate theory is utilized to establish the size-dependent integral-differential governing equations and the associated boundary conditions of viscoelastic nanoplate. The developed model accounts for the viscoelastic behavior of the linear non-aging materials using integral-type constitutive relations through the Boltzmann's principle of superposition. Finally, an analytical solution is derived for the simple supported plates according to the aforementioned phenomena using the Navier method and Laplace transformation. In the context of the linear viscoelasticity, a comprehensive parametric study is developed to present the influences of the various material and geometrical parameters on the bending behavior of the viscoelastic nanoplate.

1. Introduction

In the recent years, micro and nanostructures such as micro and nanobeams, plates and sheets play a prominent role in micro and nanoelectromechanical systems (MEMS and NEMS) due to their superior mechanical, electrical, chemical, thermal, and electronic characteristics, [14,21,67]. It has been experimentally demonstrated that size-dependent effects are important and has major impacts on the behavior of micro-/nano-structures, [7,19,30,50,65]. Experimentally-detected size-dependent behavior of micro/ nanostructures cannot be predicted using classical continuum theories. Hence, several nonclassical higher order continuum mechanics theories have been developed to capture the size effect in which constitutive equations that involve additional material constants as well as the classical Lame's constants. Among these theories, the strain gradient theory [57,58], couple stress theory [55,56,64], micropolar theory [3,5], nonlocal continuum theory [1,2,4,6], modified strain gradient theory [19], modified couple stress theory [22] and surface elasticity theory [46,47]. In the following, bending behavior of elastic/viscoelastic nanoplates based on the modified couple stress and surface elasticity theories will be discussed briefly.

Among all size-dependent continuum theories, the modified couple stress theory (MCST) that proposed by Yang et al. [22] has the advantage of including only one additional material length scale parameter to capture the size effect of microstructures. In this theory, only the symmetric part of the couple stress tensor is used as the suitable measure for the continuum micro-rotation. In recent years, many researchers derived formulations for the mechanical analysis of the elastic micro and nanoplates and investigate their bending behavior based on this theory. Among them are Tsiatas [26], Jomehzadeh et al. [20], Ma et al. [28], Asghari [36], Akgöz and Civalek [8], Thai and Kim [29], Shaat et al. [39] and Jung and Han [66].

One of the main features of nanoscale materials and structures is the high ratio of the surface to the volume of the continuum and, consequently, the surface energy effects cannot be ignored. However, both the atomistic simulations and the experimental evaluations have strongly indicated that the surface to volume ratio plays a critical role in nano-sized problems, [35,60]. Gurtin and Murdoch [46,47] proposed a nonclassical continuum theory which incorporates the effects of surface energy. Gurtin-Murdoch surface elasticity theory has been widely accepted and employed in the literature to investigate the surface energy effects on the mechanical behavior aspects of nanos-

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tructures for its computational efficiency and versatility. Several studies have been dealing with the size-dependent bending response of elastic micro and nanoplates based on Gurtin-Murdoch surface elasticity theory, Lim and He [9], Lu et al. [51], Huang [15], Lü et al. [11], Wang and Wang [33], Shaat et al. [37,38], Ansari et al. [52], Liu and Rajapakse [10], Zhang et al. [25], Attia et al. [41], Gao and Zhang [68] and Sapsathiarn and Rajapakse [72].

It is important to take into account the simultaneous effects of the microstructure and surface energy on the behavior of micro/nanostructures. However, few works have been developed for investigation the behavior of nanoplates, incorporating the simultaneous effects of microstructure and surface energy. Recently, Shaat et al. [39] considered the combined effects of surface energy and microstructure in the context of the MCST in investigating the bending behavior of Kirchhoff nanoplate. This model was extended by Mahmoud and Shaat [24] to the bending behavior of functionally graded ultra-thin Mindlin plates. Circular Kirchhoff plates subjected to axisymmetric loading were modeled by Zhang et al. [25] based on the MCST and surface energy effect. A nonclassical model for rectangular Kirchhoff nanoplate, based on the Hamilton’s variational formulation, was developed by Gao and Zhang [68]. Their developed model exploited both, the MCST and surface elasticity theory to investigate the behavior of the nanoplates. Recently, Attia and Mahmoud [44] proposed an integrated nonlocal couple-stress elastic continuum model, incorporating the surface energy effect. This developed model has been used to investigate simultaneously, the effects of long range interatomic interactions, microstructure local rotation and surface energy effect on the bending and buckling behaviors of Bernoulli-Euler nanobeam.

Regarding the investigation of nonclassical formulations of the quasistatic and dynamic responses of viscoelastic nanostructures, Gonzalez-Lopez and Fernandez-Saez [61] investigated the bending vibrations of nonlocal Euler-Bernoulli beams, resting on viscoelastic damping patches. The damping behavior of the patch was represented by spatial kernel and relaxation functions and the derived governing equations were solved using the method presented by Lei et al. [69]. Friswell et al. [48] investigated the dynamics of nonlocal viscoelastic beams using finite element method. Lei et al. [70,71,72] investigated the influence of the nonlocality on the free vibration of viscoelastic damped Euler-Bernoulli and Timoshenko nanobeams using Kelvin-Voigt viscoelastic model and velocity-dependent external damping. Zhang and Fu [31] proposed a nonclassical model for Euler-Bernoulli viscoelastic microbeam based on the MCST to predict the quasistatic and dynamic responses of electrically actuated microbeams. Pouresmaeeli et al. [62] studied the free vibration characteristics of nonlocal viscoelastic orthotropic Kirchhoff nanoplate resting on a Kelvin-Voigt viscoelastic foundation. Karličić et al. [17,16] analyzed the free transverse vibration of a nonlocal viscoelastic Kirchhoff multi-nanoplate system embedded in a viscoelastic medium and the free longitudinal vibration of a nonlocal viscoelastic double-nanorods that coupled continuously by a light viscoelastic layer. Their models were based on Kelvin-Voigt viscoelasticity and D’Alembert’s principle. Ansari et al. [53,54] studied the linear and nonlinear free vibrations, respectively, of a fractional viscoelastic Timoshenko nanobeam using the concept of fractional derivative and nonlocal elasticity theory. Recently, Attia and Mahmoud [43] developed a nonlocal couple-stress viscoelastic continuum model for Bernoulli-Euler viscoelastic nanobeam, based on generalized Hamilton’s principle. A new analytical solution was presented for the quasistatic viscoelastic bending response of the nonclassical nanobeam, with different boundary conditions. Attia [45] and Attia and Mohamed [40] developed a nonlinear model of electrically actuated viscoelastic Euler-Bernoulli microbeams based on the modified couple stress theory. The proposed model accounted for system nonlinearities including the axial residual stress, geometric nonlinearity, electrical forcing with fringing effect, Casimir force and van der Waal forces. The authors developed a new generalized differential/integral quadrature method to solve the resulting governing equation with various boundary conditions.

Considering the behavior of nanoscale continuum, taking into account the effect of surface energy in the framework of viscoelasticity, a few works are reported. Ru [13] suggested a dissipative surface stress model to study the effect of surface dissipation on the vibration behavior of elastic nanobeams. His model was an extension of the viscoelastic Zener model in the presence of an initial surface tension. Altenbach et al. [27] formulate the effect of surface viscoelasticity on the effective properties of nanosized thin-walled elastic structures. Expressions of the stress resultant tensor were derived using the extension of the Gurtin–Murdoch model without considering the effect of the surface residual stress.

Although few models for size-dependent viscoelastic nanobeams and nanoplates have been developed in the aforementioned studies based on the nonlocal theory, modified couple stress theory or surface energy effect, no study dealt with the combined effects of surface energy and microstructure on mechanical behavior of nanoscale viscoelastic plates. To the best of authors' knowledge, viscoelastic microstructure local rotation and viscoelastic surface effects on the behavior of viscoelastic nanoplates have not been investigated up to now. Hence, in the present paper, a new variational size-dependent viscoelastic micro/nanoplate model is developed, including the effects of microstructure couple stress, surface energy and surface residual stress. Gurtin-Murdoch surface elasticity theory and the modified couple stress theory are, respectively, employed and extended to include the effects of surface energy and microstructure rotation in the framework of linear viscoelastic theory. The viscoelastic constitutive equations of the bulk and surface are expressed by a Boltzmann superposition integral. The couple stress tensor is obtained incorporating measures for the elastic and the viscous behaviors of the nanoplate. Also, the surface stress tensor is derived depending on the elastic and viscous surface parameters. Based on the Kirchhoff plate theory and the D’Alembert’s principle, the integral-differential equation of motion and boundary conditions have been derived. A new closed form solution for the deflection of simply-supported viscoelastic nanoplate is obtained utilizing the Navier’s approach and Laplace transform. The presented analytical solution is verified with those available in the literature and an excellent agreement is obtained. Finally, the influences of the material length scale parameter, surface viscoelastic modulus, residual surface stress, relaxation time, delayed to initial extensional elastic modulus ratio, thickness of the nanoplate, length-to-thickness ratio and aspect ratio on the bending response of the viscoelastic nanoplate are elucidated in the context of viscoelasticity.

2. Formulation of the mathematical model

In this section, the governing equation and corresponding nonclassical boundary conditions of a viscoelastic Kirchhoff micro/nanoscale plate are exactly derived using the D’Alembert’s principle. The modified couple stress and Gurtin-Murdoch surface elasticity theories are extended and modified to capture the microstructure couple stress, and surface energy effects, in the context of viscoelasticity. For this purpose, consider a thin rectangular plate with length a, width b and uniform thickness h as shown in Fig. 1. A Cartesian coordinate system $\boldsymbol{x}\equiv(x, y, z)$ is introduced so that the upper (top) and lower (bottom) surfaces S^+ and S^- of the plate are defined by $z = +h/2$ and $z = -h/2$, respectively, and the (x, y)-plane lies in the mid-plane of the undeformed plate. For a Kirchhoff plate, the displacement field of an arbitrary point in a plate can be expressed as follows

$$u_x(x, y, z, t) = -zw_{,x}(x, y, t), \; u_y(x, y, z, t) = -zw_{,y}(x, y, t) \quad \text{and} \quad u_z(x, y, z, t) = w(x, y, t) \tag{1}$$

where u_x, u_y and u_z are respectively, the x-, y- and z-components of the displacement vector $\boldsymbol{u}$ and $w(x, y, t)$ is the lateral deflection of the mid-plane of the nanoplate. A comma followed by a subscript denotes