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Anisotropy and plastic flow in the circular bulge test

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ABSTRACT

The procedures for obtaining the stress *vs.* strain curve from the circular bulge test are investigated in detail resorting to finite element analysis. Particular attention is given to in-plane anisotropic materials for which remains a lack of knowledge about the distributions near the pole of the bulge specimen of variables such as the surface radius of curvature, sheet thickness, principal stresses and strains as well as stress and strain paths. This study seeks to understand and evaluate the errors inherent to the commonly used experimental procedure for assessing the hardening curve from the bulge test. The procedure assumes that the stress path at the pole is equibiaxial. An empirical equation relating the stress path with the strain path at the pole of the cap is suggested to improve the determination of the biaxial stress *vs.* strain curve, which holds particular prominence in cases of strongly anisotropic sheets.

1. Introduction

Sheet metal forming processes are demanded to manufacture components for the automotive, aeronautics and other industries. The finite element method (FEM) is commonly used nowadays for simulating and optimizing sheet metal forming processes. However, the numerical simulation results are dependent on the convenient characterization and modelling of the mechanical behaviour of metal sheets. Whatever the constitutive model used in the simulations (*i.e.* hardening law and anisotropic yield criterion), the strategies for identifying its parameters as well as the experimental tests and procedures used in the analysis play an important role in the characterization of the metal sheets mechanical behaviour [1–8]. The parameters of the models are generally determined with recourse to tensile and other experimental tests, such as shear, cruciform and bulge [9].

The circular bulge test under hydraulic pressure allows achieving relatively high strain values before necking and enables the definition of the hardening law for a wide range of plastic deformation [10]. The periphery of the metal sheet is immobilized through a drawbead, which prevents the peripheral region of the sheet from moving into the radial direction [11–13]. Then, a hydraulic pressure is applied on the inner surface of the sheet, promoting an approximately spherical shape in the region near the pole of the cap and inside a circle of constant latitude [14,15]. Under these conditions, a biaxial stress path occurs at the pole of the cap.

For evaluating the stress *vs.* strain curve, the evolutions of pressure, radius of curvature and strain at the pole of the cap should be recorded during the test. The measurements of radius of curvature and strain can be performed by a spherometer and an extensometer, respectively [16,17]. An optical system can replace these mechanical systems with advantages, since it enables the description of the geometry and strain distributions on the sheet surface during the bulge test [18,19]. In both cases, the membrane theory that relates the stresses at the pole with the pressure, radius of curvature and sheet thickness must be used [20].

The analysis of the bulge test results, including the application of the membrane theory, still presents uncertainties, despite of the recent recommended procedure by ISO 16808 (2014) [21]. In fact, the accurate evaluation of the stress *vs.* strain curves depends on assumptions and simplifications, whose assessment are still under study. For example, in a recent study Mulder et al. [22] examined the validity and the conditions for using the membrane theory, which includes issues related to the geometry of the cap that affects the evaluation of the radii of curvature near the pole and the equibiaxial stress state assumption in case of in-plane anisotropic materials. They showed that the spherical function can be successfully replaced by the ellipsoid function, for describing the bulge surface up to large distances from the pole of the cap, in order to estimate the curvature radius. This allows increasing the data to be considered without loss of accuracy, since it enables the reduction of the scatter. In case of an in-plane anisotropic material, Mulder et al. [22] concluded that the test conditions force the material towards an equibiaxial strain state and the deviations of the average stress from the equibiaxial stress are less than 3%, for an in-plane

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anisotropic material described by the Hill'48 criterion, with values of the anisotropy coefficient, r, for angles of 0, 45 and 90°, such that $r_0=0.5$ and $r_{45}=r_{90}=1.0$ [23]. Yoshida [24] estimated the stress and strain paths during the bulge test, in case of in-plane anisotropic materials, using finite element analysis. He concludes that the stress path at the pole of the cap deviates from equibiaxial between 1–5%, depending on the degree of anisotropy of the materials. Also, he observed that the deviation from unity of the ratio between the curvature radii of the cap along the rolling and the transverse directions is less than 0.4%, for equivalent plastic strains up to 0.6, and less than 2%, for equivalent plastic strains up to 1.0. However, these results only concern materials with relatively low anisotropy in the sheet plane.

The current work presents a numerical study on the circular bulge test of metal sheets, performed with the *DD3IMP* in-house finite element code [25–27]. It examines the geometry and the stress and strain distributions near the pole of the cap. This analysis also concerns the relationship between stress and strain paths. Materials with anisotropy in the sheet plane are particularly considered. The methodology for the experimental determination of the stress *vs.* strain curve of metal sheets and associated errors is also analysed. The Hill'48 criterion [28] and the Swift law [29] are used due to their simplicity, but other constitutive models are also tested.

2. Numerical modelling and analysis

In this section, the numerical model of the circular bulge test is defined and the methodology for the evaluation of the biaxial stress *vs.* strain curve is described.

2.1. Modelling

The geometry of the tools considered in the test is schematically shown in Fig. 1, where $R_M=75$ mm is the die radius, $R_1=13$ mm is the die profile radius, $R_D=95$ mm is the radius of the central part of the drawbead and $R_S=100$ mm is the initial blank radius of the circular sheet. This geometry was built based on the experimental bulge test used by Santos et al. [30]. The tools were described using Bézier surfaces, considering only one quarter of the geometry due to the material and geometrical symmetry conditions. However, in order to simplify the analysis, the drawbead geometry was neglected and its effect was replaced by a boundary condition imposing radial displacement restrictions on the nodes placed at a distance equal to R_D from the centre of the circular sheet [13]. The contact with friction was described by the Coulomb law with a friction coefficient of 0.02 [31]. The numerical simulations were carried out with the *DD3IMP* in-house code [25–27] assuming an incremental increase of the pressure applied to the sheet inner surface. The blank sheet, 1 mm thick, was discretized with linear 8-node solid elements, using two layers of elements through the thickness. More details about the spatial discretization adopted are given in [32,33].

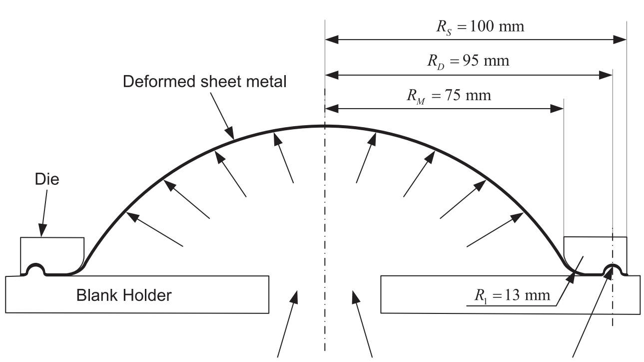

Fig. 1. Bulge test, with the identification of the principal dimensions of the tool according to Santos et al. [30].

The constitutive model adopted for the finite element analysis assumes that: (i) the elastic behaviour is isotropic and described by the generalised Hooke's law (with the value of the Young's modulus, $E=210$ GPa, and the Poisson's ratio, $\nu=0.30$, in all cases); (ii) the plastic behaviour is described by the orthotropic Hill'48, Drucker + L or CB2001 yield criteria and the hardening model by the Swift or the Voce isotropic laws.

The Hill'48 yield surface is described by the equation [28]:

$$F(\sigma_{yy}-\sigma_{zz})^2+G(\sigma_{zz}-\sigma_{xx})^2+H(\sigma_{xx}-\sigma_{yy})^2+2L\tau_{yz}^2+2M\tau_{xz}^2+2N\tau_{xy}^2 = Y^2, \tag{1}$$

where σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, τ_{xz} and τ_{yz} are the components of the Cauchy stress tensor, defined in the principal axes of orthotropy, and F, G, H, L, M and N are the material parameters describing the anisotropy of the metal sheet. Y represents the yield stress and its evolution during deformation $Y=f(\bar{\varepsilon}^{\mathrm{p}})$.

The Hill'48 yield criterion was chosen because of its simplicity, but its lack of flexibility does not allow it to correctly describe some anisotropic behaviours, including those designated by Banabic as ‘first and second order anomalous’ behaviours [34]. In this context, the Drucker + L and the CB2001 criteria were also chosen due to their degree of flexibility.

The Drucker + L and the CB2001 yield criteria [35] are extensions of the Drucker isotropic yield criterion [36]. The Drucker + L yield criterion is described by the equation:

$$\left[\frac{1}{2}\mathrm{tr}(\mathbf{s}^2)\right]^3 - c\left[\frac{1}{3}\mathrm{tr}(\mathbf{s}^3)\right]^2 = 27\left(\frac{Y}{3}\right)^6, \tag{2}$$

where tr($\mathbf{s}$) is the trace of the stress tensor $\mathbf{s}$, resulting from the linear transformation of the Cauchy stress tensor, $\boldsymbol{\sigma}$, and c is a weighting isotropy parameter, ranging between $-27/8$ and $9/4$, to ensure the convexity of the yield surface. When c equals zero, this criterion coincides with the Hill’48 yield criterion. The $\mathbf{s}$ stress tensor is given by:

$$\mathbf{s} = \mathbf{L}:\boldsymbol{\sigma}, \tag{3}$$

where $\mathbf{L}$ is the linear transformation operator proposed by Barlat et al. [37]:

$$\mathbf{L} = \begin{bmatrix} (C_2+C_3)/3 & -C_3/3 & -C_2/3 & 0 & 0 & 0 \\ -C_3/3 & (C_3+C_1)/3 & -C_1/3 & 0 & 0 & 0 \\ -C_2/3 & -C_1/3 & (C_1+C_2)/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_6 \end{bmatrix}, \tag{4}$$

in which C_i, with $i=1, \ldots, 6$, are the anisotropy parameters; $C_1=C_2=C_3=C_4=C_5=C_6$ for the full isotropy condition. This yield criterion includes one more parameter, the parameter c, when compared to Hill’48 yield criterion, thus being more flexible. So, when the parameter c is not zero, Hill'48 criterion cannot fully describe the behaviour of a material that follows the Drucker + L criterion.

The CB2001 yield criterion is given by the equation:

$$(J_2^0)^3 - c(J_3^0)^2 = 27\left(\frac{Y}{3}\right)^6, \tag{5}$$

where J_2^0 and $J^0{}_3$ are the second and third generalised invariants of the Cauchy stress tensor: