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Abstract

With the aim to answer to the question about what is the correct notion for curvature to adopt in constitutive relationships, this paper considers a geometrically exact beam theory built on very basic kinematic assumptions. The theory is developed in a consistent way, by deducing equation of motions from the Principle of Virtual Work. Further, by stipulating a relation between internal work for one- and three-dimensional beams, relationships among internal forces and stresses are found. Constitutive equations, which end up to be strongly coupled and nonlinear, are written in explicit form for a specific material model with linear behavior. The role of different notions of curvature to adopt in analysis of beams is investigated and linear, linearized and nonlinear equations for bending moment are considered. In particular, uncoupled linear approximation provides indication about the more suitable curvature definition. Further, a mechanical interpretation of generalized internal stresses is also given and benchmark numerical examples enlighten some features of the model.

Keywords: geometrically exact beam theories, curvature, large displacements, large rotations, lagrangian formulation

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1. Introduction

When a beam is bent in a plane without shearing deformation, two different cross sections exhibit different rotations, in general. Let us consider cross sections at a mutual reference distance which is approaching zero and measure the variation of rotation with respect to the undeformed element. If the length of the element remains unchanged during the deformation process, the derivative of the rotation with respect to the reference configuration coincides with the local curvature. It is called as the normalized [1], flexural[2] or mechanical [3, 4] curvature.

If the bending is accompanied by stretching, the curvature is expressed, from a geometric point of view, by the mechanical curvature times the inverse of the stretch. Let us call it as the geometric curvature, adopting the nomenclature introduced in [2], [4] and [5].

Notice that the mechanical curvature is defined as the material measure of curvature in [6] and, accordingly, the geometric curvature is denoted as the spatial measure of curvature in [5].

In formulating a constitutive relationship for the bending moment, a key question is which between material or spatial is the correct measure to adopt, especially in case the beam undergoes large deflections and rotations.

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