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Locating design point in structural reliability analysis by introduction of a control parameter and moving limited regions

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ABSTRACT

In reliability analysis, computation of reliability index and finding design point is still a challenge. In this paper a new efficient reliability algorithm to locate design point is proposed. The proposed algorithm takes benefit from two significant means in its efficient search for the design point. One means is an updating rule by which the candidate of design point is updated and moved towards real design point. The criteria of updating in this rule are designed such that the candidate moves on an effective general path towards real design point. The other means is the introduction of a control parameter by which the search process at each iteration is limited to a relatively small region. This parameter controls the candidate of design point on its defined general path and does not let it leave the path. These two means have made the proposed algorithm very reliable in finding design point. Through numerical examples the accuracy and efficiency of the proposed algorithm is shown.

1. Introduction

In structural analysis uncertainty can be observed in material, load and geometric properties. Thus, in order to have a realistic understanding of the behavior of a structure, uncertainty has to be taken into account [1–3]. Structural reliability theory is the tool to bring the effect of uncertainty into the analysis procedure [4–6]. In reliability analysis, the probability of failure P_f is to be evaluated by the following multi-dimensional integral

$$P_f = \int_{g(X)<0} f_X(X)dX \tag{1}$$

where $X=[X_1,X_2,\ldots,X_n]^T$ represents the vector of random variables and $f_X(X)$ is the joint probability density function (JPDF) of this vector. $g(X)$ is limit state function (LSF) such that $g(X) > 0$ and $g(X) < 0$ define safety and failure domain, respectively. However, the evaluation of the aforementioned integral is very difficult, especially in the large and complex structures or structures with low probability of failure and implicit LSFs. The reason of this difficulty lies in involving multiple integral and JPDF of random variables. That is why more attention has been attracted to other alternatives such as approximation methods [7–10] and simulation methods [11–14].

Approximation methods, among which first- and second-order reliability method (FORM and SORM) are the most important ones, try to approximate LSF by Taylor expansion at design point or most probable point (MPP). As Hasofer and Lind [15] have defined, this point located on limit state surface has the minimum distance from the origin of standard normal coordinate system or U-space (in which mean and standard deviation of all variables are 0 and 1, respectively). The distance of design point to the origin has been called reliability index denoted by β. Generally, approximation methods are suitable for some examples but they require a differentiable LSF and this condition is unattainable in many cases. Besides, in the cases of highly nonlinear LSFs they may converge very slowly or even result in divergence.

In simulation methods such as Monte Carlo simulation (MCS), random samples are generated using a sampling density function (SDF) and LSF is evaluated for each sample. Probability of failure is considered the proportion of the number of samples corresponding to failure (negative value of LSF) to total sampling number. MCS does not need mathematical form and derivatives of the LSF but the large number of simulations, needed to increase the accuracy, is a big computational burden. That is why many researches have been focused on reducing the number of required samples to make it more practical [16–19]. Importance sampling (IS) is a useful approach which focuses on selecting a more efficient SDF, than the original JPDF of random variables, for generating random samples [20–22].

In this paper a new algorithm for locating design point, in a step-by-step and purposeful way, is proposed. The proposed algorithm assigns a candidate to search for the real design point. The process is carried out by specifying a general path for the movement of the candidate. The

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contribution of an MPP updating rule is to be applied in specifying such a path. Besides, the process forces the candidate to stay in a limited region around the path. This is done using a newly defined control parameter. The parameter, indicating the dimension of the limited region, plays a significant role in making the answers reliable. The significance of the control parameter is more considerable in highly nonlinear cases, where the answers are sensitive to the relatively small movements. It will be shown that the MPP candidate, which is also called so-far MPP (i.e. the best answer until the current iteration), moves in its general path limited by control parameter.

2. General concepts

It should be noted that the MPP updating rule, used in the proposed algorithm for movement towards real MPP, has been represented by the authors [23]. Before going through the details of this MPP updating rule and the proposed algorithm in Section 3, the MPP updating rule represented by Jahani et al. [18] is explained in the current section. This section clarifies why the rule presented in Section 3 has been selected to be used in the proposed algorithm.

By the definition of Hasofer and Lind [15], MPP is a point on limit state surface having the minimum distance from the origin of *U*-space. That is why Jahani et al. pick a newly generated point as the new MPP if the point (1) has a smaller distance from the origin of *U*-space, compared to the previous MPP and (2) has smaller absolute value of g, again compared to the previous MPP [18]. These conditions are schematically displayed in Fig. 1 where the newly generated point replaces previous MPP because it satisfies both aforementioned conditions.

This approach may work in many problems but it has a big drawback. To explain this drawback consider Fig. 2 and suppose the MPP in this figure is such close to limit state surface that $g(MPP)$ has become very close to zero. In the figure the newly generated point is illustrated, too. As it is seen, this point is a better candidate because it is closer to real MPP. But since this point is not as close to limit state surface as the previous MPP (i.e. since the second condition is not satisfied), the point fails to become the new MPP. Thus, one potentially good candidate of design point is missed because of the deficiency and inflexibility of the conditions. This drawback, which may create difficulties in the movement of so-far MPP, has been removed by the authors [23] in the way explained in Section 3.2.

3. Proposed algorithm

In the proposed algorithm the main purpose is to locate design point through a step-by-step and reliable approach. The so-called limited region and the applied MPP updating rule are two important components in the proposed algorithm. Thus, they are to be explained in detail in Sections 3.1 and 3.2.

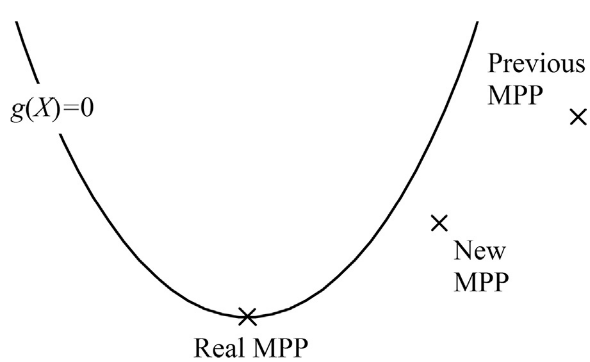

Fig. 1. Schematic description of Jahani's MPP updating rule.

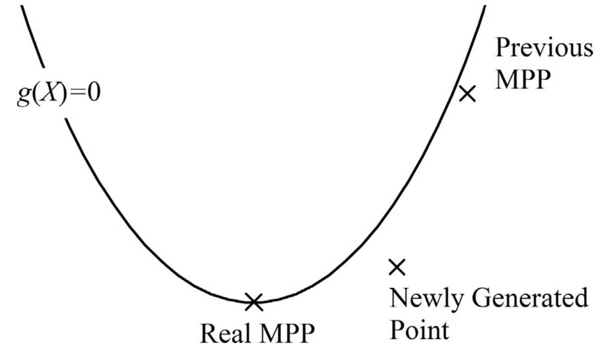

Fig. 2. Schematic description of the drawback of Jahani's MPP updating rule.

3.1. Limited region for generation of random samples

In this section, we are supposed to focus on one specific iteration and draw a region in which the SDF is allowed to generate the corresponding sample of that iteration. Limiting such a region for generating samples can ensure reaching the vicinity of real MPP. In the following, characteristics of the region and significance of the so-called control parameter are presented separately.

3.1.1. Characteristics of the limited region

We define a hypercube in *X*-space as the aforementioned allowable region with all its edges parallel to the coordinate axes. The so-far MPP is selected as the center of this region. Dimension of the hypercube is also determined using the following two points as the end points of one diagonal

$$X_{\min} = MPP - s \times \sigma$$
$$X_{\max} = MPP + s \times \sigma \quad (2)$$

where MPP and σ are the vector of so-far MPP and the vector of standard deviation of random variables, respectively, both in the original *X*-space. The positive scalar s is a predetermined constant called control parameter and is half of the dimension of the hypercube. Having these two points and MPP, the hypercube is specified in *X*-space. Eq. (2) implies that $X_{\min}$ is obtained from MPP using negative increments in all n directions and $X_{\max}$ is obtained in the same way but using positive increments (n is the number of random variables). Thus, the ith component of $X_{\min}$ and $X_{\max}$ ($i=1{:}n$) are the lower and upper bounds of ith random variable, respectively. In order to force the SDF to generate ith random variable x_i between its corresponding bounds (i.e. $x_{i,\min} < x_i < x_{i,\max}$), the following relation must be satisfied

$$F_{X_i}(x_{i,\min}) < F_{X_i}(x_i) < F_{X_i}(x_{i,\max}) \quad (3)$$

where $F_{X_i}(x_i)$ is the marginal cumulative distribution function associated with ith variable in SDF. Thus if a random value, corresponding to $F_{X_i}(x_i)$, is selected between the boundary values of Eq. (3), not commonly between 0 and 1, x_i falls between $x_{i,\min}$ and $x_{i,\max}$ and the generated point falls inside the limited hypercube. It should be noted that since the purpose of sample generation in the proposed algorithm is to find a point better than the so-far MPP, not to find the distribution of LSF, it does not make any difference how the SDF is distributed. Thus, for simplicity and without loss of generality and regardless of the real distribution of variables, the applied SDF in generating samples is assumed to have normal distribution located on the so-far MPP and with the same standard deviation vector as the random variables.

According to the above explanations, at each iteration the end points of Eq. (2) can be obtained using the so-far MPP and known constant s. Since the hypercube is specified by the end points, a random point can be generated inside the hypercube at this iteration and using the explanations of Eq. (3). After that, the point is evaluated to see whether or not it is better than the so-far MPP. The criteria for this evaluation will be provided by the MPP updating rule of Section 3.2. If