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Strengthening and design of the linear hardening thick-walled cylinders using the new method of rotational autofrettage

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ABSTRACT

The new approach introduced for the autofrettage called the rotational autofrettage is applied here for thick-walled cylinders made of materials with linear strain hardening behavior. The concept of the rotational autofrettage originates from the fact that large enough angular velocity in a thick-walled cylinder causes beneficial residual stress which is the prerequisite of the autofrettage process. For this purpose, elasto-plastic stress distribution in a rotating thick-walled cylinder are obtained analytically for both loading and unloading phases using Tresca's yield criterion and considering the bauschinger effect. In the following, the residual stress distribution are obtained to determine the best level of autofrettage for strengthening and design of thick-walled cylinder prior to industrial use with the aim of increasing strength-to-weight ratio. It is concluded that rotational autofrettage causes better results in comparison with the conventional method, pressure autofrettage. Moreover, rotational autofrettage method is used to obtain the best dimensionless thickness for a cylinder to withstand high internal pressure.

1. Introduction

Thick-walled cylinders are one of the most applicable equipment which are used, due to their high pressure bearing capability, for maintenance and transformation of high pressure fluids in nuclear, oil and petrochemical industries; in addition to weaponry and military industries. For the vast application of these equipment many researchers have studied the strengthening and design process of them. Since the design process based on elastic stress analysis leads to increase in the amount of used material, the researchers are attracted to the methods based on elastoplastic stress analysis for design of thick-walled cylinders such as autofrettage method. For the most probability of fracture at the inner radius of high pressurized thick-walled cylinders, creating beneficial residual stress at this area, which is the base of autofrettage method, helps strengthening the cylinder. Autofrettage method consists of a cycle of loading and unloading phase with plastic deformation in the material which finally creates appropriate compressive residual stress near the bore. When the cylinder is subjected to high pressure in the working condition, the resulted tensile stress should first overcome the residual stress caused by autofrettage and then create tension at the bore, which means more pressure bearing capacity of cylinder. Elastoplastic loading is the prerequisite of residual stress creation. A good autofrettage process depends on accuracy of predicted residual stress and the determination of appropriate required level of autofrettage.

To find the appropriate level of autofrettage the stress distribution in both loading and unloading phase and subsequently the residual stress should be specified. Several investigations have been done to determine the elastoplastic stress distribution in a pressurized thick-walled cylinder. Saint-Venant [1] was the first one who raised the autofrettage concept by determining the mathematical relations for plastic stress distribution in a pressurized thick-walled cylinder. Bland [2] using Tresca's yield criterion and its associated flow rule determined the stress and strain relations in loading and unloading phase explicitly for a pressurized thick-walled cylinder considering the impact of hardening behavior of material and heat temperature. Chen [3] suggested a model which could present a more accurate description of high strength steel materials while neglecting the hardening during loading phase but considering the bauschinger and hardening effect during unloading. He determined a closed-form solution for the residual stress distribution in a pressurized thick-walled cylinder also using the Tresca's criterion and its associated flow rule. Orcan [4] considering elastic-perfectly plastic model and Tresca's yield criterion with its associated flow rule determined the stress and deformation distribution for the plane strain state in a solid cylinder subjected to internal heat generation and then he [5] obtained the residual stress and deformation distribution for a solid cylinder with same condition for the states of being partly and fully plasticized. Gulgec and Orcan [6] considered the

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yield stress as a function of temperature and obtained the plastic stress and strain in a thick-walled cylinder subjected to heat source. Livieri and Lazzarin [7] obtained an analytical solution to determine the residual stresses in an autofrettaged cylindrical pressure vessel considering the von-mises yield criterion and assuming the bauschinger effect as a constant. They discussed the effects of both strain hardening and $\sigma - \varepsilon$ curve on the residual stresses. Huang [8] introduced a general autofrettage model based on actual $\sigma - \varepsilon$ curve of material and considering Von-Mises yield criterion and bauschinger effect as a constant parameter presented an analytical solution for residual stress distribution in plain strain state. This model had a more accurate estimation of material behavior in comparison with non-hardening model and was appropriate for materials with different strain hardening. Hojjati and Hassani [9] also used Von-Mises yield criterion and considering elastic unloading obtained the optimum autofrettage pressure explicitly for plain strain state in thick-walled cylinders made of materials with strain hardening behavior. They showed that it could be used with a good accuracy for plain stress state. Darijani et al. [10] obtained an analytical solution for the residual stress distribution in a pressurized thick-walled cylinder considering Tresca's yield criterion and linear strain hardening model and assumed the bauschinger effect as a function of tensile pre-strain. They described a reasonable method to determine the best level of autofrettage in order to strengthen and design a thick-walled cylinder subjected to high internal pressure. Zheng and Xuan [11] considering power law strain hardening material model studied autofrettage and shakedown of thick walled cylinders both theoretically and with finite element method. They presented a closed form solution of limit loading and optimum autofrettage pressure for an open ended cylinder subjected to thermo-mechanical loads. They concluded that the autofrettage process has no effect on the shakedown behavior after several load cycles. In addition, they [12] studied the shakedown of perforated thick walled cylinders and investigated the related impact of radial opening size.

Jahromi et al. [13] used an extension of variable material property (VMP) method to evaluate the residual stresses in an autofrettage thick vessel made of FGM. Gao et al. [14] employed unified yield criterion and introduced a solution for the autofrettage of thick-walled cylinders subjected to internal pressure and their shakedown limit. Davidson et al. [15] introduced another method of autofrettage for thick-walled cylinders made of high strength steel, named swaging method. Mahmoudi et al. [16] and Correa et al. [17] investigated two other methods of strengthening the surface of materials based on creating beneficial residual stress in them, named shot peening and laser shock processing; respectively. Xuan et al. [18] studied the time-dependent deformation in multi-material systems. The deformation due to autofrettage is negligible and it can be said that the dimensions don't change, so that it should be noted as one of significant impacts of this engineering method of design [19].

The stresses induced in rotating cylinders have been studied by many researchers. Prescott [20] determined the elastic stress distribution in both solid and hollow cylinders. Lenard and Haddow [21] obtained the angular velocity in which the cylinder undergoes plastic deformation assuming elastic-perfectly plastic model and Tresca's yield criterion. Gamer et al. [22] considering Tresca's yield criterion and elastic-perfectly plastic model obtained elastoplastic stress distribution in a rotating solid cylinder with fixed ends. Aleksandrova [23] considering elastic-perfectly plastic material model and Mises yield criterion with its associated flow rule obtained stress-displacement solution for a solid rotating disk made of homogeneous material.

Zare and Darijani [24] introduced the rotational autofrettage method for improving pressure bearing capacity and design of such cylinders using Tresca's yield criterion and elastic-perfectly plastic model. In the more recent research [24], it was proved that neglecting hardening behavior of materials enables the design process for the pressures less than one (normalized pressure); also most of materials treatment in the plastic zone shows almost linear $\sigma - \varepsilon$ relation. In this paper, the rotational autofrettage is investigated in a thick-walled cylinder while the material strain hardening beahavior is taken into account. Improving pressure bearing capacity and designing process of thick-walled cylinders with the aim of exceeding pressure-to-weight ratio are the main works done here. Therefore, linear strain hardening model is employed to develop rotational autofrettage method and make it more applicable.

2. Formulation for loading phase

A thick-walled rotating cylinder with inner radius of 'a' and outer radius of 'b' is studied while there isn't any external stresses on its inner and outer surfaces and the radius ratio ($\frac{b}{a}$) is named 'β'. In loading phase, the angular velocity increases gradually from zero to 'ω'. Since the problem is axially symmetric, polar coordinates are employed and r, θ, z are the principle directions. With the assumptions mentioned the equilibrium equation takes the form comes bellow:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \rho_0 r\omega^2 = 0 \tag{1}$$

And the compatibility equation is:

$$\frac{d\varepsilon_\theta}{dr} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0 \tag{2}$$

where σ_r, σ_θ, ω, ρ_0, ε_r and ε_θ refer to radial stress, hoop stress, angular velocity, the materials density, total radial and hoop strain, respectively. The total strain can be decomposed into the elastic and plastic parts as follows:

$$\begin{cases} \varepsilon_r = \varepsilon_r^{(e)} + \varepsilon_r^{(p)} = \frac{1}{E}[(1-\nu^2)\sigma_r - \nu(1+\nu)\sigma_\theta] + \varepsilon_r^{(p)} \\ \varepsilon_\theta = \varepsilon_\theta^{(e)} + \varepsilon_\theta^{(p)} = \frac{1}{E}[(1-\nu^2)\sigma_\theta - \nu(1+\nu)\sigma_r] + \varepsilon_\theta^{(p)} \end{cases} \tag{3}$$

where superscripts (e) and (p) denote the elastic and plastic parts of strain. To have a general solution, the following assumptions are made:

$$S_r = \frac{\sigma_r}{\sigma_0}, \quad S_\theta = \frac{\sigma_\theta}{\sigma_0}, \quad \rho = \frac{r}{a}, \quad \beta = \frac{b}{a}, \quad \in_r = \frac{E\varepsilon_r}{\sigma_0}, \quad \in_\theta = \frac{E\varepsilon_\theta}{\sigma_0},$$
$$S_e = \frac{\sigma_e}{\sigma_0}, \quad P_w = \frac{p_w}{\sigma_0}, \quad \Omega = \frac{\rho_0 a^2\omega^2}{\sigma_0}, \quad \in^{(p)} = \frac{E\varepsilon^{(p)}}{\sigma_0} \tag{4}$$

where σ_0, σ_e, p_w and E are the initial yield stress, the equivalent stress, working pressure and young modulus, respectively. So the previous Eqs. (Eqs. (1)–(3)) changes to the following ones:

$$\frac{dS_r}{d\rho} + \frac{S_r - S_\theta}{\rho} + \rho\Omega = 0 \tag{5}$$

$$\frac{d\in_\theta}{d\rho} + \frac{\in_\theta - \in_r}{\rho} = 0 \tag{6}$$

$$\begin{cases} \in_r = (1-\nu^2)S_r - \nu(1+\nu)S_\theta + \in_r^{(p)} \\ \in_\theta = (1-\nu^2)S_\theta - \nu(1+\nu)S_r + \in_\theta^{(p)} \end{cases} \tag{7}$$

If the cylinder doesn't experience plastic deformation, the stress distribution will be [24]:

$$S_r^{(e)} = \frac{-C_1}{2\rho^2} - \frac{\rho^2\Omega(3-2\nu)}{8(1-\nu)} + C_2 \tag{8}$$

$$S_\theta^{(e)} = \frac{C_1}{2\rho^2} + \frac{-\rho^2\Omega(1+2\nu)}{8(1-\nu)} + C_2 \tag{9}$$

$$S_z^{(e)} = \nu(S_r^{(e)} + S_\theta^{(e)}) \tag{10}$$

And using the boundary conditions ($S_r(1) = 0$ and $S_r(\beta) = 0$), the constants C_1 and C_2 are [24]:

$$C_1 = \frac{\Omega\beta^2(2\nu-3)}{4(\nu-1)}, \quad C_2 = \frac{\Omega(\beta^2+1)(2\nu-3)}{8(\nu-1)} \tag{11}$$