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On the analytical and finite element solution of plane contact problem of a rigid cylindrical punch sliding over a functionally graded orthotropic medium

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ABSTRACT

This study investigates the plane frictional contact problem of a cylindrical punch on a functionally graded orthotropic medium (FGOM). Both analytical and computational methods are developed to obtain the contact stresses. The elastic modulus is assumed to vary exponentially and the principal axes of orthotropy are assumed to be aligned with the global coordinates. In the analytical formulation, plane elasticity equations are converted to Cauchy-type singular integral equation of the second type by using Fourier Transform technique. The resulting integral equations are then solved numerically to obtain the contact stresses throughout the medium for plane stress condition. In the computational model, the elastic modulus of each finite element is specified at its centroid by using the predefined functional variation. The results obtained from the finite element analysis is verified by comparing the results obtained using the analytical formulation. The results of this study can guide tribology engineers in determining the contact stresses that have a great effect on the wear resistance of mating components.

1. Introduction

Contact mechanics is a key subject in designing machine elements such as brakes, clutches, internal combustion engines, bush and ball bearings, hinges, gaskets and is of major interest in modern manufacturing methods [45]. The motivation for using graded materials goes back to the time when Japanese blacksmiths used a graded transition from a hardened edge to a softer core [50]. Because of their smooth transition in elastic and thermal material properties, Funtionally Graded Materials (FGMs) have attracted attraction as wear resistant or thermal barrier coatings in applications such as diesel engine piston heads, heteroepitaxial multilayers used in semiconductor devices, quantum wells and light-emitting diodes. An important study on the wear resistance capability of FGMs was conducted by Suresh [53] who concluded that graded materials suppress indentation cracks during normal and frictional sliding.

The research on the mechanics of FGMs was started in the mid-1980 s with a space craft project in Japan to alleviate the shortcomings of conventional and homogeneous composite materials. The aim of the project was to develop super heat resistant materials that can withstand high temperature gradients observed in aerospace applications [38]. Since then the application and utilization of FGMs has expanded to applications such as; functionally graded cemented carbide tool [39], nanostructured and functionally graded cathodes for solid-oxide fuel cells [35], biomedical applications [44] and piezoelectric bending devices [51]. FGMs are also used to attenuate problems resulting from fatigue, corrosion and fracture and to improve the tribological performance of contacting mating parts in an assembly such as strongly adherent super hard amorphous carbon films [52], to improve fracture toughness, biaxial bending strength, and wear resistance in joint prostheses Mishina et al. [36], to improve the adhesion of DLC films to silicone substrates [26], functionally graded diamond-like carbon coatings [8] and to improve wear resistance [57,58]. It has been shown in the literature that FGMs can suppress the formation of herringbone cracks under sliding frictional contact [54] and can eliminate conical cracking resulting from Hertzian indentation [27,42,43].

Because of the processing methods used in the manufacturing of graded coatings, FGMs engender a highly anisotropic structure (e.g. lamellar microstructure with cleavage planes parallel to the boundary when the plasma spraying technique is used [46,49] and a columnar structure with cleavage planes perpendicular to the boundary when the physical vapor deposition technique is used [28,48]).

There are significant studies on the contact mechanics of graded isotropic coatings or graded half-planes. Suresh and his co-workers

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studied the indentation of solids with graded elastic properties. Specifically, they considered axisymmetric indentors [18] and point force [19] for the load transfer between mating parts. Normal sliding and rolling type of contact were addressed by Giannakopoulos and Pallot [17] and they provided an analytical closed-form solution using power law variation in elastic modulus. In order to investigate the load displacement behavior on the graded half-space, power law and error function grading were considered by Lee et al. [34]. Frictional sliding contact problems for graded isotropic coatings having an exponential variation in thickness direction for various punch profiles are investigated by Guler and Erdogan [20–22]. Ke and Wang [29,30] considered the same problem solved by Guler and Erdogan [20] using a multi-layered model for the contact mechanics of FGMs. The shear modulus of each layer was assumed to vary linearly in the multi-layered model. The results obtained from both sets of authors are in excellent agreement with each other. Later, Yang and Ke [55] added a homogeneous top layer to the FGM coating-substrate system and investigated the frictionless contact problem by using a rigid circular indenter. The graded layer is assumed to be composed of arbitrary number of layers with piecewise linear shear moduli. Chidlow and Teodorescu [5] considered the two-dimensional contact mechanics of inhomogeneously elastic solids split into three distinct layers. Chidlow et al. [7] developed a new model to approximate the contact pressure and contact half-width by using Saint-Venant's principle. They investigated the sub-surface stress field and the effect of coating thickness on material failure. Later, they proposed a semi-analytical model for frictional contact problem, where the material properties of the graded elastic transition layer exhibit an exponential variation [6]. There are studies the literature that possess elastic gradation in material properties not only in the thickness direction but also in the lateral direction. For example, Dag et al. [10] considered the contact mechanics of laterally graded materials loaded by rigid flat and triangular shaped indenters. In the following study by the same authors, the same problem was considered using circular indenter profiles [11]. In a recent study, Alinia et al. [2] considered a fully coupled problem where a rigid cylinder is in sliding contact with an FGM coating. They presented both surface and subsurface stress distribution and compared these results with those obtained by King and O'Sullivan [31] when the coating is homogeneous.

Although there are quite a lot of studies on the contact mechanics of graded isotropic materials, the studies on the functionally graded orthotropic medium (FGOM) are rather limited owing to the increase in elastic constants in the constitutive relations. In isotropic materials it is sufficient to describe the constitutive relation with two elastic constants however, four elastic constants are required for the constitutive equation in plane stress conditions and seven constants are required for plane strain conditions in orthotropic materials. As a result of this the governing equations become more complex and depend on the orthotropic material constants. For example, Guler [24] obtained an analytical solution to the sliding contact problem of a homogeneous orthotropic medium. With soil mechanics applications in mind, Bakirtaş [4] considered the frictionless contact problem of a rigid punch indenting an elastic orthotropic half-plane.

The present study is a continuation of the work conducted by Kucuksucu et al. [33]. In this paper, a cylindrical punch profile that is approximated by a parabolic function is studied. The contact mechanics problem is solved using two different techniques. The first one is an analytical technique based on singular integral equations (SIEs), and the second one is a numerical technique that depends on the finite element method. The analytical technique requires Naviers equations that are derived employing the elements of plane orthotropic elasticity. Using the Fourier transformation techniques, the contact mechanics problem is reduced to a Fredholm singular integral equation of the second type which is solved numerically by using an expansion-collocation technique. The results obtained using analytical technique are compared with finite element analyses. There is a perfect agreement between these two sets of results. The presented results illustrate the influences of orthotropic material constants, the coefficient of friction and the non-homogeneity parameter on the surface tractions as well as the subsurface stresses.

The analytical solution of the problem is described in Section 2. The direction of the material property gradation is taken as perpendicular to the contact surface. The graded orthotropic half-plane is loaded with a cylindrical punch that is sliding with a constant friction coefficient. The resulting SIE is solved using the expansion-collocation technique. The implementation of the finite element method for the contact analysis of the graded orthotropic materials is described in Section 3. The results obtained using the analytical approach are compared with the results of the finite element for validation and described in Section 5. It is observed that the analytical and the numerical techniques developed in the current study lead to highly accurate results. The main results provided the effect of orthotropic material properties, coefficient of friction and non-homogeneity parameter on the contact pressure distribution and in-plane stresses at the contact surface (see Section 6). The concluding remarks are provided in Section 7.

2. Analytical formulation of the contact problem

2.1. Integral equation of the cylindrical punch problem

The plane contact problem between a cylindrical punch over a FGOM is shown in Fig. 1a where the principal axes of orthotropy are aligned with the (x_1, x_2) coordinate system. It is assumed that the friction coefficient is spatially constant and obeys Coulombs friction law (i.e., $Q = \eta P$), where η is the coefficient of static friction. Let u_i and σ_{ij} $(i, j = 1, 2)$ denote the displacement and stress components and E_{ii}, G_{ij} and ν_{ij} $(i, j = 1, 2, 3)$ denote material's elastic parameters. Using the the following definitions, four independent engineering constants E_{11}, E_{22}, G_{12}, ν_{12} are replaced by the following four parameters namely; the effective stiffness (E), the effective Poissons ratio (ν), the stiffness ratio (δ), and the shear parameter (κ) [9,32]:

$$E = \sqrt{E_{11}E_{22}}, \quad \nu = \sqrt{\nu_{12}\nu_{21}}, \quad \delta^4 = \frac{E_{11}}{E_{22}} = \frac{\nu_{12}}{\nu_{21}}, \quad \kappa = \frac{E}{2G_{12}} - \nu. \tag{1}$$

for generalized plane stress conditions. For plane strain conditions Eq. (1) must be replaced by:

$$E = \sqrt{\frac{E_{11}E_{22}}{(1-\nu_{13}\nu_{31})(1-\nu_{23}\nu_{32})}},$$
$$\nu = \sqrt{\frac{(\nu_{12}+\nu_{13}\nu_{32})(\nu_{21}+\nu_{23}\nu_{31})}{(1-\nu_{13}\nu_{31})(1-\nu_{23}\nu_{32})}}, \delta^4 = \frac{E_{11}}{E_{22}}\frac{1-\nu_{23}\nu_{32}}{1-\nu_{13}\nu_{31}}, \quad \kappa = \frac{E}{2G_{12}} - \nu. \tag{2}$$

Using the aforementioned parameters, the relationship between the strain and the stress components can be expressed as:

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} = \frac{1}{E(x_1, x_2)} \begin{bmatrix} \delta^{-2} & -\nu & 0 \\ -\nu & \delta^2 & 0 \\ 0 & 0 & 2(\kappa+\nu) \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}, \tag{3}$$

with $(\nu_{ij}/E_{ii}) = (\nu_{ji}/E_{jj})$. Using the following definitions

$$x = \frac{x_1}{\sqrt{\delta}}, \quad y = x_2\sqrt{\delta}, \quad u(x, y) = \sqrt{\delta}\,u_1(x_1, x_2), \quad v(x, y) = \frac{1}{\sqrt{\delta}}u_2(x_1, x_2), \tag{4}$$

$$\sigma_{xx}(x, y) = \sigma_{11}(x_1, x_2)/\delta, \quad \sigma_{yy}(x, y) = \delta\sigma_{22}(x_1, x_2), \quad \sigma_{xy}(x, y) = \sigma_{12}(x_1, x_2). \tag{5}$$

Using Eqs. (4), (5), the Hooke's law can be written as

$$\sigma_{xx}(x, y) = \frac{E^*(x, y)}{1-\nu^2}\left\{\frac{\partial u(x, y)}{\partial x} + \nu\frac{\partial v(x, y)}{\partial y}\right\} \tag{6}$$