International Journal of Mechanical Sciences 120 (2017) 62–69

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Free vibration analysis of an axially moving multiscale composite plate including thermal effect

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ARTICLE INFO

Keywords:
Multiscale composite
Axially moving plate
Thermal loading

ABSTRACT

On the basis of the frequency-temperature equivalence principle and the elastic–viscoelastic equivalence, a linear mathematical model in the form of the equilibrium state equation of the moving multiscale composite plate is derived in the complex frequency domain. To describe thermomechanical properties of the plate material, viscoelastic properties of multiscale fiber reinforced stationary composites presented in literature are taken into account. Fractional standard rheological model of the plate material as the function of reduced frequency depended on the temperature is determined. Numerical investigations of free vibrations are carried out for the 0.1 wt%, 1 wt% multiscale fiber reinforced composite plates and the neat fiber reinforced composite plate in the temperature range 35–200 °C. The effects of transport speed, internal damping and the volume fraction of fibers in the plate material on natural frequencies are presented. In the range of low temperatures the values of critical transport speeds of the tested plates to a small extent differ from each other and significantly decrease with the temperature increase. In higher temperatures the critical transport speeds of the tested plates differ from each other and with the temperature increase reach constant values.

1. Introduction

Axially moving flat composite objects at high speeds, can be found in many differing technical applications. Paper webs during production, processing and printing, textile webs during processing and flat objects moving at high speeds in space are examples of thin, flat objects moving axially at high speeds. Manyfactors influence the dynamic behavior of the moving plants. The most important are: transport speed, tension, material properties and influence of external environment. Among the external factors affecting the dynamics, temperature plays an important role. Accurate identification of the temperature effect on the dynamics of the axially moving plant is important to properly control these very often expensive devices. A literature review on dynamics of axially moving systems can be found, for example, in the work Marynowski and Kapitaniak [16].

Dynamic studies in the field of composite are carried out for a long time. At the turn of the centuries composite materials began to appear particularly in theaerospace, maritime and space industries because they offer a number of advantageous mechanical properties. These properties include resistance to electrochemical corrosion, high strength and rigidity with less weight than conventional materials. The state of researches in the dynamics of laminated composite and sandwich plates in the paper by Sayyad and Ghugal [19] is presented.

Carbon nanotube (CNT)-reinforced/fiber/polymer composites, called as multiscale composite, are new generation of advanced composite materials. Weight reduction characterizes these materials in many applications. Therefore, dynamic investigation of multiscale composites is of very importance. Processing, characterization and thermomechanical modeling of carbon nanotube-reinforced composites are presented in the paper by Green et al. [3]. In the paper by Kim et al. [9] the Halpin-Tsai equations [4] and woven fiber micromechanics were combined to capture the geometrical configuration of the woven structure and to integrate the nano-modified matrix properties into the multiscale composite properties. Fig. 1 shows a block diagram for evaluating the elastic properties of the CNT-reinforced multiscale composites [9].

Zhou et al. [22] analyzed the static and free vibration of carbon nanotube-reinforced composite plates using finite element method with first-order shear deformation plate theory. Bhardwaj et al. [2] developed the nonlinear flexural and dynamic response of carbon nanotube-reinforced laminated composite plates. They employed fast converging finite double Chebyshev polynomials to solve the equations and concluded that with increasing CNTs percentage and aspect ratio of CNTs, non-dimensional transverse central deflection of CNT reinforced polymer composite plate decreases. Modeling and nonlinear stress analysis of piezo laminated multiscale composite plates under a combined mechanical and electrical loading was presented by Rafiee et al. [18]. They considered symmetrically and perfectly bonded

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http://dx.doi.org/10.1016/j.ijmecsci.2016.11.013
Received 2 August 2016; Received in revised form 7 November 2016; Accepted 14 November 2016

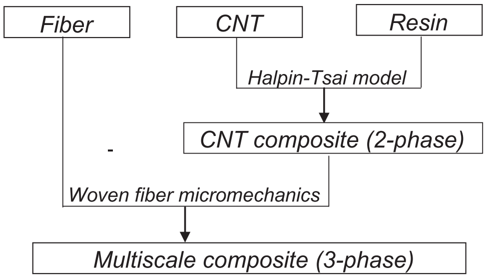

Fig. 1. Layout for computation of mechanical properties of multiscale composites.

piezoelectric layers on the top and bottom surface of the composite host and investigated the influence of the applied constant voltage and weight percentage of CNT on the deflection and stress analyses of the piezoelectric plate.

In contrast to the case of stationary composites, the literature on dynamic analysis of axially moving two-dimensional viscoelastic systems is rather limited. Zhou and Wang [23,24] studied vibration characteristics of axially moving viscoelastic rectangular plate taking into account a parabolically plate thickness. Three studies by Hatami et al. [5,6] and Marynowski [15] devoted to free vibrations of axially moving multi-span composite plates, viscoelastic Navier-type plate, and viscoelastic Levy-type plate, respectively were published in recent decade. To model visoelastic properties of moving plate, two parameters Kelvin-Voigt and three parameters standard rheological models were used in these papers. Tang and Chen in the paper [20], and Yang et al. [21], studied vibrations, bifurcations and chaos of axially moving viscoelastic plates using finite difference and non-linear model for transverse displacements. They paid attention on bifurcations and chaos but also studied the dynamic characteristics of a linearized elastic model with the help of eigenfrequncy analysis. The state of researches in the field of axially moving viscoelastic plates in the book by Banichuk et al. [1] is presented.

Important branch of current researches on dynamics of axially moving plants are studies of axially moving micro/nanoscale structures, such as subminiature belts or other nanoscale components and molecular machines. In such systems, the size effects should be taken into consideration and classical theories are incapable to describe their dynamic behavior. Then as an side-dependent theoretical approach, the nonlocal theories are used to model axially moving micro/nanostructures. For example the nonlocal stress gradient theory was applied to investigate dynamics of nanoscale beams (Lim et al.[10,11], Liu et al. [13]).

A great variety of composite materials, and also a trade secret hiding the full results of the research carried out by the manufacturers, make it necessary to conduct fundamental research in dynamics field. The literature review shows also that the dynamic studies of axially moving multiscale composite plants including thermal effects have not been taken.

The objective of the current paper is to conduct free vibration analysis of the axially moving multiscale composite plate taking into account thermal effects. On the basis of the frequency-temperature equivalence principle and the elastic–viscoelastic equivalence, a linear mathematical model in the form of the equilibrium state equation of the moving orthotropic multiscale composite plate is derived in the complex frequency domain. To describe thermomechanical properties of the plate material, viscoelastic properties of multiscale fiber reinforced stationary composites presented in literature are taken into account. Fractional standard rheological model of the plate material as the function of reduced frequency depended on the temperature is determined. The effects of temperature, transport speed, internal damping and the volume fraction of fibers in the plate material on natural frequencies are analyzed.

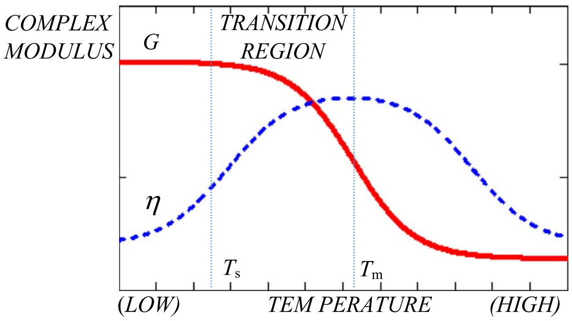

Fig. 2. Effect of temperature on complex modules behavior.

2. The effects of temperature and frequency

When a viscoelastic material is subjected to periodic load, the induced deformation is also periodic out of phase. Then the stress-strain relationship can be characterized by complex modulus given in the frequency domain by

$$E_v(\omega) = E(\omega)(1 + i\,\eta(\omega))\,, \tag{1}$$

where $E(\omega)$ is the Young's modulus, and $\eta(\omega)$ is the loss factor.

It is well known that most polymeric materials exhibit dynamic behavior which depends strongly on frequency and temperature. Polymers are composed of long intertwined and cross-linked molecular chains, each containing very many atoms. The internal molecular interactions which occur during vibration leads to energy dissipation and damping. If the polymers are homogenous and isotropic, the stiffness and damping characteristics vary with temperature and frequency. The shear and extensional moduli are closely related to each other for homogenous and isotropic polymers [7].

The complex modules properties of polymers vary strongly with temperature, in ways particular to each polymer composition. Fig. 2 illustrates nonlinear behavior of some typical polymers.

Fig. 2 shows that above softening temperature T_s in the transition region, the shear modulus decreases rapidly and the loss factor rises to a maximum in the temperature T_m and then falls again. In temperatures above the transition region, the modulus is low, and as the temperature continues to rise, the material disintegrates.

While the effects of frequency are small for typical metal solid materials, the effects of frequency is much stronger for many polymers. The effect of frequency is the inverse of the effect of temperature, increasing frequency is similar to the effect of decreasing temperature, but at much different rates, as Fig. 3 illustrates. The difference is very significant. While the temperature may vary by a few hundred degrees to reach the transition region, the corresponding change of frequency encompass many orders of magnitude. In this range the frequency can vary from 10^{-8} Hz to 10^{8} Hz or more. For low frequencies the loss factor and shear modulus increase slightly. In transition region one can

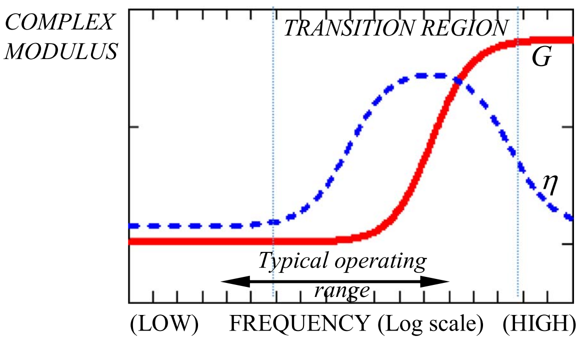

Fig. 3. Effect of frequency on complex modules behavior.