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A two-stage time domain subspace method for identification of nonlinear vibrating structures

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ABSTRACT

To identify multiple degree-of-freedom vibrating structures with local nonlinearities, a two-stage time domain approach based on the subspace method is proposed in this study. The locally nonlinear system to-be-identified is divided into an underlying linear part described by the FRF and a local nonlinear part described by nonlinear coefficients. The identification process of the proposed approach is not the same as the existing single-stage method. It identifies the underlying linear system before the local nonlinearities. The proposed approach reduces the dimensions of matrices which are used to calculate the system's state-space model and also gives a more appropriate estimation of the order of the underlying linear system with the utilization of classical spectrum estimation techniques. Both numerical and experimental examples are given to verify the performance of the method. Results show that the method is more accurate and reliable than the single-stage method, especially in a noisy environment.

1. Introduction

In the past few decades, lots of effort has been put into the field of identification of nonlinear structural systems and various kinds of methods have been proposed. Some developed techniques include the restoring force surface method, the Volterra series, the time series analysis, the Kalman filter based method [1–11]. Among the various kinds of nonlinear structures, there is a common type that the nonlinearities of the system are caused by several local nonlinearities. For example, the joint connecting different substructures is a major source of the local nonlinearity with features such as friction, gaps, stick-slip behavior. The assumption of local nonlinearities simplifies the decoupling of linear and nonlinear parts of the system and thus makes it possible to utilize some well-developed linear techniques.

Among the various kinds of approaches is one group of methods based on reverse path formulation. These methods separate the whole system into underlying linear part and nonlinear part and are especially suitable for nonlinear systems with local nonlinearities. Bendat [12] firstly introduced the reverse path (RP) spectral method to identify single degree-of-freedom (DOF) systems and then Rice and Fitzpatrick [13] developed the method to identify multiple DOF systems. In order to overcome the limitations of the RP method such as requirement of excitation location, a more complex conditioned reverse path (CRP) method was proposed by Richards and Singh [14]. Since the formulation of the CRP method is complicated and causes more computational complexity, Adams and Allemang [15] put forward the nonlinear identification through feedback of the output (NIFO) method and Magnevall et al. [16] proposed another modified RP method to alleviate the computational burden. Zhang et al. [17] developed a forward selection reverse path method to locate spatial nonlinearities in multiple DOF system. Most of the RP methods are frequency domain methods and are based on the spectrum analysis techniques. The orthogonalized reverse path (ORP) method, as a time domain method, proposed by Muhamad et al. [18] can be treated as a counterpart of the frequency domain CRP method. Another time domain method called the nonlinear subspace identification (NSI) method was proposed by Marchesiello and Garibaldi [19] and some comparisons with the NIFO method are also demonstrated in their study. Noel et al. [20] compared the time domain and frequency domain subspace-based methods based on a nonlinear spacecraft study. Haroon and Adams [21] also used the time and frequency domain nonlinear system characterization to identify mechanical fault. Moaveni and Asgarieh [22] applied the subspace identification method to identify nonlinear structures by treating them as time-varying linear systems.

The NSI method overcomes the drawback that the estimations of nonlinear coefficients and frequency response function (FRF) of the underlying linear system are less accurate near the resonance zone because of the increased signal correlation [19]. Also, as a time domain method, it depends less on the accuracy of the spectral analysis

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techniques. The NSI method is essentially based on the stochastic subspace method [23] which is a valuable technique especially for identification of linear system. The stochastic subspace method is established on the discrete state-space model and utilizes projection of matrix and singular value decomposition to enhance the method's robustness and is suitable for identification of multiple-input multiple-output (MIMO) system in noisy environment. Similar to other frequency domain RP methods, the NSI method also treats the whole system as an underlying linear system with several internal nonlinear restoring forces and then reverses the input-output path to formulate state-space models. The method utilizes properties of the derived formulation of the FRF of the underlying linear system and calculates the local nonlinear coefficients under the assumption that the mass matrix, damping matrix and stiffness matrix of the system are all symmetrical matrices.

In the present study, a two-stage time domain method based on stochastic subspace algorithm is proposed. The method identifies the linear part and the nonlinear part of the whole system in two sequential stages and thus needs two measurements under different levels of excitation. The method is more accurate and reliable benefitting from a more suitable way to determine the order of the state-space model than a single-stage edition especially in noisy environment. Also, estimating the system's matrices in two steps can alleviate possible numerical problems such as calculating matrices with large dimensions. In Section 2, the stochastic subspace method is simply introduced. In Section 3, the proposed two-stage method is described and in Section 4, numerical and experimental examples are given to illustrate the correctness of the analysis and the effectiveness of the method. Also, some comparisons are made to demonstrate the performances of the two-stage method compared with the single-stage method.

2. Stochastic subspace method

The stochastic subspace method, an identification method for discrete linear state-space models, contains a group of algorithms [23–26] which utilize the projection of matrix and the singular value decomposition techniques. In this section, the numerical algorithms for the subspace state-space system identification (N4SID) [23] are given briefly.

The purpose of the N4SID is to identify the discrete-time deterministic stochastic state-space model with measured values of inputs and outputs. The identified model is represented as

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_d\mathbf{x}_k + \mathbf{B}_d\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k = \mathbf{C}_d\mathbf{x}_k + \mathbf{D}_d\mathbf{u}_k + \mathbf{v}_k \end{cases} \tag{1}$$

where $\boldsymbol{x}_k$, $\boldsymbol{u}_k$, $\boldsymbol{y}_k$ denote the state vector, the input vector and the output vector at the discrete time t_k, respectively. $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ respectively represent the process and measurement noises and are all assumed to be the zero-mean Gaussian white noises. $\mathbf{A}_d$, $\mathbf{B}_d$, $\mathbf{C}_d$ and $\mathbf{D}_d$ are the matrices to-be-identified of the discrete-time state-space model. Iterating the two equations in Eq. (1) yields

$$\begin{aligned} y_k &= \mathbf{C}_d\mathbf{A}_d{}^{k-1}x_1 + \mathbf{C}_d\mathbf{A}_d{}^{k-2}\mathbf{B}_d u_1 + \mathbf{C}_d\mathbf{A}_d{}^{k-3}\mathbf{B}_d u_2 + \ldots + \mathbf{C}_d\mathbf{B}_d u_{k-1} \\ &\quad + \mathbf{C}_d\mathbf{A}_d{}^{k-2}w_1 + \mathbf{C}_d\mathbf{A}_d{}^{k-3}w_2 + \ldots + \mathbf{C}_d w_{k-1} + \mathbf{D}_d\mathbf{u}_k + v_k \end{aligned} \tag{2}$$

For identifying the matrices $\mathbf{A}_d$ and $\mathbf{C}_d$, a so-called extended observability matrix $\boldsymbol{\Gamma}$ and two block Toeplitz matrices $\boldsymbol{\Theta}$ and $\boldsymbol{\Xi}$ are defined as

$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{C}_d \\ \mathbf{C}_d\mathbf{A}_d \\ \vdots \\ \mathbf{C}_d\mathbf{A}_d{}^r \end{bmatrix} \tag{3}$$

$$\boldsymbol{\Xi} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_d & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_d\mathbf{A}_d & \mathbf{C}_d & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{C}_d\mathbf{A}_d{}^{r-2} & \mathbf{C}_d\mathbf{A}_d{}^{r-3} & \cdots & \mathbf{C}_d & \mathbf{0} \end{bmatrix} \tag{4}$$

$$\boldsymbol{\Theta} = \begin{bmatrix} \mathbf{D}_d & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C}_d\mathbf{B}_d & \mathbf{D}_d & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_d\mathbf{A}_d{}^{r-2}\mathbf{B}_d & \mathbf{C}_d\mathbf{A}_d{}^{r-3}\mathbf{B}_d & \cdots & \mathbf{D}_d \end{bmatrix} \tag{5}$$

Then $\boldsymbol{y}_k$ in Eq. (2) at different discrete times can be rearranged as

$$\mathbf{Y} = \boldsymbol{\Gamma}\mathbf{X} + \boldsymbol{\Theta}\mathbf{U} + \boldsymbol{\Xi}\mathbf{W} + \mathbf{V} \tag{6}$$

where

$$\begin{cases} \mathbf{Y} = [\bar{y}_1 \;\; \bar{y}_2 \;\; \cdots \;\; \bar{y}_N], & \bar{y}_k = [y_k{}^T \;\; y_{k+1}{}^T \;\; \ldots \;\; y_{k+r-1}{}^T]^T \\ \mathbf{X} = [\bar{x}_1 \;\; \bar{x}_2 \;\; \ldots \;\; \bar{x}_N], & \bar{x}_k = [x_k{}^T \;\; x_{k+1}{}^T \;\; \ldots \;\; x_{k+r-1}{}^T]^T \\ \mathbf{U} = [\bar{u}_1 \;\; \bar{u}_2 \;\; \ldots \;\; \bar{u}_N], & \bar{u}_k = [u_k{}^T \;\; u_{k+1}{}^T \;\; \ldots \;\; u_{k+r-1}{}^T]^T \\ \mathbf{W} = [\bar{w}_1 \;\; \bar{w}_2 \;\; \ldots \;\; \bar{w}_N], & \bar{w}_k = [w_k{}^T \;\; w_{k+1}{}^T \;\; \ldots \;\; w_{k+r-1}{}^T]^T \\ \mathbf{V} = [\bar{v}_1 \;\; \bar{v}_2 \;\; \ldots \;\; \bar{v}_N], & \bar{v}_k = [v_k{}^T \;\; v_{k+1}{}^T \;\; \ldots \;\; v_{k+r-1}{}^T]^T \end{cases} \tag{7}$$

in which N and r denote the user-defined integers which satisfy $r > 2n$ and $N >> 2n$ (n is the order of the state-space model in Eq. (1)).

To remove the latter three terms in the right side of Eq. (6), the matrix projection and the instrumental variables techniques are utilized. Specifically, right multiplication of a projection matrix $\prod_{\mathbf{U}^T}^{\perp}$ and a matrix $\mathbf{P}^T$ results in

$$\mathbf{Y}\prod\nolimits_{\mathbf{U}^T}^{\perp}\mathbf{P}^T = \boldsymbol{\Gamma}\mathbf{X}\prod\nolimits_{\mathbf{U}^T}^{\perp}\mathbf{P}^T \tag{8}$$

where

$$\prod\nolimits_{\mathbf{U}^T}^{\perp} = \mathbf{I} - \mathbf{U}^T(\mathbf{U}\mathbf{U}^T)^{-1}\mathbf{U} \tag{9}$$

$$\mathbf{P} = [\mathbf{U}_P{}^T \;\; \mathbf{Y}_P{}^T]^T \tag{10}$$

in which $\mathbf{U}_P$ and $\mathbf{Y}_P$ are constructed similarly as the matrices $\mathbf{U}$ and $\mathbf{Y}$, and the relationship $(\boldsymbol{\Xi}\mathbf{W} + \mathbf{V})\prod_{\mathbf{U}^T}^{\perp}\mathbf{P}^T = \mathbf{0}$ can be guaranteed by using the previous measured data [27]. To increase the accuracy of the subsequent singular value decomposition, two full rank matrices $\mathbf{H}_1 = (\mathbf{P}\prod_{\mathbf{U}^T}^{\perp}\mathbf{P}^T)^{-1}(\mathbf{P}\mathbf{P}^T)^{1/2}$ and $\mathbf{H}_2 = \mathbf{I}$ [28] are added into Eq. (8) and the following equation is derived

$$\mathbf{H}_1\boldsymbol{\Gamma}\mathbf{X}\prod\nolimits_{\mathbf{U}^T}^{\perp}\mathbf{P}^T\mathbf{H}_2 = \mathbf{H}_1\mathbf{Y}\prod\nolimits_{\mathbf{U}^T}^{\perp}\mathbf{P}^T\mathbf{H}_2 \tag{11}$$

It can be concluded from Eq. (8), through some rank analysis, that the column space of $\boldsymbol{\Gamma}$ is the same with the column space of $\mathbf{Y}\prod_{\mathbf{U}^T}^{\perp}\mathbf{P}^T$. Therefore, through the singular value decomposition, the estimated observability matrix $\boldsymbol{\Gamma}^e$ and the true observability matrix $\boldsymbol{\Gamma}$ have the same column space and can be transformed by right multiplying a nonsingular matrix $\mathbf{T}$ as

$$\begin{cases} \mathbf{H}_1\mathbf{Y}\prod_{\mathbf{U}^T}^{\perp}\mathbf{P}^T\mathbf{H}_2 = \mathbf{Q}\mathbf{S}\mathbf{V}^T \\ \boldsymbol{\Gamma} = \boldsymbol{\Gamma}^e\mathbf{T} = \mathbf{H}_1{}^{-1}\bar{\mathbf{Q}}\mathbf{T} \end{cases} \tag{12}$$

where $\mathbf{S}$ is the diagonal matrix in singular value decomposition and $\bar{\mathbf{Q}}$ is the matrix obtained by removing some columns from $\mathbf{Q}$ according to the values of their corresponding diagonal elements in $\mathbf{S}$. Then the estimated matrices $\mathbf{A}_d{}^e$ and $\mathbf{C}_d{}^e$ can be computed from $\boldsymbol{\Gamma}^e$. $\mathbf{C}_d{}^e$ is the first s lines of $\boldsymbol{\Gamma}^e$ in which s represents the dimension of output $\boldsymbol{y}_k$. $\mathbf{A}_d{}^e$ can be calculated by the shift invariance method [28] with