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An improved fast plane wave expansion method for topology optimization of phononic crystals

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ABSTRACT

Phononic crystals (PnCs) are artificially made materials composed of periodically arranged structures capable of manipulating acoustic/elastic wave propagation characteristics. In this paper, an improved fast plane wave expansion method (IFPWEM) is developed to obtain the band structures of PnCs. In the method presented, the continuity of the algorithm has been improved by eliminating the jump discontinuity points as well as decreasing the number of wave vectors used. Implementing these changes results in increased computational efficiency when compared to the traditional fast plane wave expansion method (FPWEM). In order to increase the band gap width produced by the PnCs, an adaptive genetic algorithm is adopted to optimize the PnCs structural topology for in-plane wave mode (*xy* mode). The numerical results yielded from optimization of two-dimension (2D) PnCs with a symmetric square lattice microstructure verifies that the efficiency of the IFPWEM is significantly greater than the conventional FPWEM and finite element methods.

1. Introduction

Phononic crystals (PnCs) are composed of periodically arranged structures that allow for the creation of band gap regions, in which the propagation of acoustic or elastic waves is completely prohibited [1–11]. Sigalas and Economou [12] firstly presented the band structures of elastic waves propagating in a two-dimension (2D) periodic fluid-solid system in 1993. Since then, there has been a great deal of work devoted to the characterization of PnC band structures. Betsabe et al. [13] presented an experimental demonstration of an omnidirectional band gap formation in a finite multilayer composite composed of two elastic layers. Then in 2006, Lin et al. [15] experimentally obtained the reflection transfer function of a phononic band gap by using a single-quantum-well in a phononic cavity. Tan [14] presented a generalized eigenproblem with a hybrid matrix method for stable analysis of Floquet wave propagation in one-dimensional (1D) PnCs in 2010. Meanwhile, Liu and Fang [16] proposed the lumped-mass method for the band structure analysis of elastic wave propagation in a 1D PnC rod. Afterwards, Yudistira et al. [17] introduced the finite element method (FEM) to demonstrate the existence of non-radiative surface acoustic wave band gaps in 2D piezoelectric phononic crystals with holes. Meanwhile, Gao et al. [18] also investigated the band gap properties of a 2D local-resonant phononic crystal by the FEM with a homogenous matrix. Cao et al. [35] investigated the singularity of the Bloch theorem in a fluid/solid phononic crystal, in which a plane wave expansion method (PWEM) was use to calculate the band structure and transmission spectrum of the air/rigid system. Later, Cao et al. [36] also proposed a revised formulation of eigenproblem for phononic crystals, which significantly improves the convergence in band structure calculations. Based on Cao's researches, Quan et al. [20] developed a plane wave expansion method using a parallel shift scheme to evaluate the band gap formation in a 2D PnC with L-shape scatters. Afterwards, the fast plane wave expansion method (FPWEM), which was proposed by Liu et al. in 2014, had been employed to investigate the band gap structures of PnCs with arbitrary or pixel scattering elements [21]. Besides the traditional pure PnCs above, the band structures of PnCs with multi-fields coupling materials, such as piezoelectric and piezomagnetic PnCs, had been investigated in [19,37–41].

In PnCs, the band structure characteristics are highly dependent on the topology of the constituent unit cells that make up the materials microstructure. The design and optimization of the PnC unit cell geometry has attracted a great deal of interest recently with the objective of creating structures capable of producing extremely wide band gap regions. Sigmund and Jensen [22] used a topology optimization method to design PnCs capable of exhibiting pure band gaps over specific frequency regions. Bilal and Hussein [23,24] presented a vacuum-soli-porous PnC, in which the band gap with a normalized width exceeded 60%. A 2D PnC with an arbitrarily asymmetric lattice

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was also obtained by combining the genetic algorithm (GA) with the FEM [25]. Zhong et al. [26] maximized the band gap width in a PnC by using a GA combined with the fast plane wave expansion method (GA-FPWEM). Meanwhile, Liu et al. [21] applied a GA-FPWEM to the topology optimization of PnCs with square lattices. A bi-directional evolutionary inspired optimization method with a penalization parameter was also developed to achieve convergent optimal solutions for structures comprised of one or multiple materials [27]. Dong et al. [28,29] used the non-dominated sorting-based GA for optimal PnC design which yielded both large band gap width and as well as a reduction in overall mass. Li et al. [30] recently developed a bi-directional evolution based structure optimization algorithm combined with the homogenization method to yield a maximum band gap width between the two adjacent bands in a square unit cell.

Although past researches have been conducted with the objective of optimizing PnC topology utilizing various iterative methods, there remains a lot of works need for further investigation and improvement. For instance, although the topology optimization of PnCs based on the FEM has high accuracy, its efficiency is extremely low which causes the optimization procedure is very time-consuming. The FPWEM is proposed to improve the calculation efficiency while guaranteeing the necessary computational accuracy. But the FPWEM also has drawbacks, which need to be eradicated. If the individual components of the unit cell are highly contrasting, the continuity of the interfacial material boundaries is poor. It means that the concurrent jump discontinuities (or the jump discontinuity points) exist at the boundaries of different materials. In order to ensure the calculation accuracy, the number of wave vectors considered for each band gap needs to be substantial in size. The need for an increase in the number of wave vectors used during computation means that the efficiency of the FPWEM is significantly reduced. In this paper, the IFPWEM is introduced for calculating band structures of PnCs. Based on the Lauren inverse theorem, By eliminating the jump discontinuity points on the boundaries of different materials, the continuity of algorithm has been improved, and the number of wave vectors used for the band structure analysis is decreased. Thus, compared with the FPWEM, the IFPWEM model can achieve a much higher computational efficiency without loss of precision. Moreover, the GA optimization process alone is not capable of generating an optimal unit cell topology using conventional settings. Thus, the iteration times of conventional GAs can be considerable. Similarly, the adaptive genetic algorithm (AGA), which can generate the optimal unit cell topology in the early stages of the optimization process, is also introduce to reduce the overall iteration times required and improves the convergence of GA. In short, an adaptive genetic algorithm is implemented alongside the improved fast plane wave expansion method (AGA-IFPWEM) in pursuit of the optimal PnC design for the in-plane elastic wave attenuation.

The rest of the paper is organized as follows. In Section 2, the IFPWEM is proposed for the band structure analysis of PnCs. The AGA-IFPWEM model for topology optimization of PnCs is detailed in Section 3. In Section 4, three numerical examples are employed to investigate the efficiency and effectiveness of the proposed 2D AGA-IFPWEM model. Conclusions are then presented in Section 5.

2. Improved fast plane wave expansion method

2.1. Elastic wave equation

Based on the elastic dynamics theory, the wave propagation in a three-dimensional inhomogeneous elastic medium is governed by

$$\rho\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial x}(\lambda\nabla\cdot\mathbf{u}) + \nabla\cdot(\mu(\nabla u_x + \frac{\partial \mathbf{u}}{\partial x})), \tag{1}$$

$$\rho\frac{\partial^2 u_y}{\partial t^2} = \frac{\partial}{\partial y}(\lambda\nabla\cdot\mathbf{u}) + \nabla\cdot(\mu(\nabla u_y + \frac{\partial \mathbf{u}}{\partial y})), \tag{2}$$

$$\rho\frac{\partial^2 u_z}{\partial t^2} = \frac{\partial}{\partial z}(\lambda\nabla\cdot\mathbf{u}) + \nabla\cdot(\mu(\nabla u_z + \frac{\partial \mathbf{u}}{\partial z})), , \tag{3}$$

where ρ is the material density, and the Lamé coefficients are denoted by λ and μ. The displacement vector is written as $\mathbf{u} = \{u_x u_y u_z\}^{\mathrm{T}}$, and its divergence is defined as $\nabla\cdot\mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$.

Assuming the propagating elastic waves are constrained within xy-plane, such that, $\partial\mathbf{u}/\partial z = 0$. The resulting elastic wave equations can then be expressed as

$$\rho\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial x}\left((2\mu+\lambda)\frac{\partial u_x}{\partial x} + \lambda\frac{\partial u_y}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x})\right), \tag{4}$$

$$\rho\frac{\partial^2 u_y}{\partial t^2} = \frac{\partial}{\partial y}\left((2\mu+\lambda)\frac{\partial u_y}{\partial y} + \lambda\frac{\partial u_x}{\partial x})\right) + \frac{\partial}{\partial x}\left(\mu(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y})\right), \tag{5}$$

$$\rho\frac{\partial^2 u_z}{\partial t^2} = \frac{\partial}{\partial x}(\mu\frac{\partial u_z}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial u_z}{\partial y}). \tag{6}$$

Combining Eq.'s (4) and (5) results in the coupled in-plane equation (governing xy mode), and Eq. (6) can be used to describe the out-of-plane equation (governing z mode).

2.2. Fast plane wave expansion method

Periodic variations of material properties, such as elastic constants and mass densities, are intrinsic properties associated with PnCs. In a PnC, the material parameters (ρ, λ and μ) are periodic functions of the spatial coordinate $\mathbf{r}$. Based on the spatial coordinate $\mathbf{r}$, the characteristic of any pixel (point location) in the unit cell can be calculated by its location in reference to the center pixel (illustrated in Fig. 1). The Fourier series expansion of material constants at arbitrary spatial locations is then obtained to quantify these spatially varying material properties. In Eq. (7) below, g denotes the material parameter as a function of spatial location. The Fourier series expansion of material constant g can be written as

$$g(\mathbf{r}) = \sum_{\mathbf{G}} g(\mathbf{G})\ e^{i\mathbf{G}\cdot\mathbf{r}}, \tag{7}$$

where $g(\mathbf{G})$ is a Fourier coefficient and can be expressed as

$$g(\mathbf{G}) = \frac{1}{S}\iint_s g(\mathbf{r})\, e^{-i\mathbf{G}\cdot\mathbf{r}} d^2\mathbf{r}, \tag{8}$$

where S and $\mathbf{G}$ are the unit cell area and the reciprocal vector, respectively.

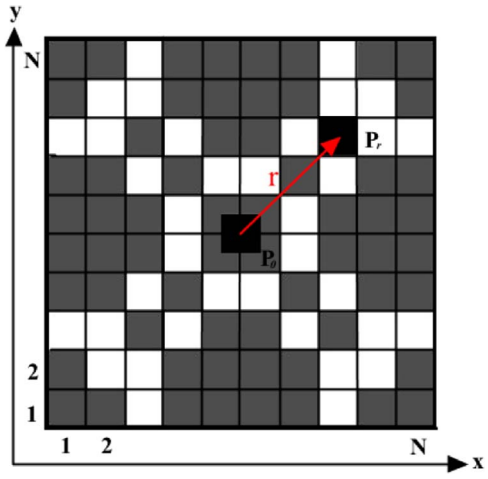

Fig. 1. The structure of phononic crystal in which the material parameter of the pixel P_r can be calculated by the center pixel P_0 and the spatial coordinate, $\mathbf{r}$. Note that in Fig. 1, the black and white represent lead and epoxy resin, respectively.