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Natural convection in a trapezoidal cavity filled with a micropolar fluid under the effect of a local heat source

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ABSTRACT

This paper analyzes laminar natural convection of micropolar fluid in a trapezoidal cavity with a local heater. The bottom and top walls of the enclosure are adiabatic while the left vertical wall and part of the right inclined wall are kept at low and high constant tempratures, respectively. The rest part of the inclined wall is adiabatic. Governing equations formulated in dimensionless variables such as stream function, linear vorticity, angular vorticity and temperature have been solved by finite difference method of the second order accuracy. Computations have been carried out to analyze the effects of Rayleigh number, Prandtl number, vortex viscosity parameter and the heater location on streamlines, isotherms and vorticity profiles as well as the variation of the average Nusselt number and fluid flow rate. It has been shown that bottom position of the heater reflects the heat transfer enhancement.

1. Introduction

The recent industrial processes are characterized by the use of new materials, which cannot be described by Newtonian fluids. Due to this reason, many non-Newtonian models have been proposed. Among these models, the micropolar fluids have been introduced by Eringen [1,2] in order to take into account the effects of local structure and micromotions of the fluid particles which cannot be described by the classical models. The incompressible micropolar fluids represent liquids consisting of rigid, randomly oriented spherical particles suspended in a viscous medium, where the deformation of fluid particles is ignored. The related mathematical model is based on the introduction of a new vector field (the microrotation), which describes the total angular velocity field of the particles rotation. Hence, one new equation is added representing the balance law of local angular momentum. There are many examples of micropolar fluids flows that are relevant for practical applications as flows of biological fluids in thin vessels, polymeric suspensions, liquid crystals, slurries, colloidal fluids, exotic lubricants, etc. (see Chiu and Chou [3]). During the last few decades, the research interest in micropolar fluid theory has significantly increased due to its enormous applications in many industrial processes. The pioneering work of Eringen [1,2] was extended in boundary layer theory by Peddieson and McNitt [4]. Peddieson [5] applied the micropolar fluid model in turbulent flow also. Extensive reviews of the theory and its applications can be found in the review papers by Ariman et al. [6,7] and in the books by Lukaszewicz [8] and Eringen [9]. It is worth pointing out here the very interesting papers by Borrelli et al. [10–13] on MHD stagnation point flow of a micropolar fluid. Some theoretical studies have been compared and favorably agree with experimental measurements (see Ariman et al. [14]). Furthermore, Kolpashchikov et al. [15] have indicated a way to measure micropolar parameters experimentally. However, more experimental and theoretical work is still required on this topic.

Further, it should be stated that during the past several decades, extensive studies on heat transfer in regular cavities and enclosures filled with a viscous (Newtonian) fluid have been done and various extensions of the problem have been reported in the literature (see Vahl Davis [16]). However, it is necessary to study the heat transfer for more complex geometries because the prediction of heat transfer for complex geometries is a topic of great importance and these surfaces often occur in many applications (see Peterson and Ortega [17]).

Motivated by the practical importance of the micropolar fluids, the main objective of this paper is to understand the fundamentals of various heating and cooling strategies, and to achieve a high performance for the free convection in a trapezoidal cavity filled with a micropolar fluid having a local heat source using the mathematical micropolar fluid model proposed by the pioneering papers of Eringen [1,2]. It should be mentioned that numerical studies on the free

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Nomenclature

g	gravitational acceleration, m·s^{-2}
H	height of the cavity, m
h	length of the heat source, m
j	micro-inertia density, $j = L^2$, m^2
K	vortex viscosity parameter, $K = \kappa/\mu$
L	length of the bottom wall, m
l	length of the upper wall, m
N	dimensionless microrotation
$\overline{N}$	dimensional microrotation, s^{-1}
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure, Pa
Pr	Prandtl number, $\mathrm{Pr} = \nu/\alpha$
Ra	Rayleigh number, $Ra = g\beta(T_h - T_c)L^3/(\alpha\nu)$
T	temperature of the fluid, K
T_c	temperature of the cooled wall, K
T_h	temperature of the hot wall, K
$\overline{u}$, $\overline{v}$	dimensional velocity components along $\overline{x}$ and $\overline{y}$ coordinates, m·s^{-1}
u, v	dimensionless velocity components along x and y coordinates
$\overline{x}$	dimensional coordinate measured along the bottom wall of the cavity, m
$\overline{y}$	dimensional coordinate measured along the vertical wall of the cavity, m
x, y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity, m^2·s^{-1}
β	volumetric expansion coefficient of the fluid, K^{-1}
γ	spin-gradient viscosity,$\gamma = (\mu + \kappa/2)j$, kg·m·s^{-1}
θ	dimensionless temperature
κ	vortex viscosity, kg·m^{-1}·s^{-1}
μ	dynamic viscosity, kg·m^{-1}·s^{-1}
ρ	fluid density, kg·m^{-3}
ψ	dimensionless stream function
ω	dimensionless vorticity

convection flow in a two-dimensional right angle trapezoidal enclosure filled with a viscous fluid or fluid-saturated porous medium has been conducted by several authors [18–26]. Iyican and Bayazitoglu [18] investigated natural convective flow and heat transfer within a trapezoidal enclosure with parallel cylindrical top and bottom walls at different temperatures and plane adiabatic side walls. A critical Rayleigh number has been presented depending on the tilting angle, where unicellular convection has been observed. Perič [19] studied natural convection in trapezoidal cavities with a series of systematically refined grids from 10×10 to 160×160 control volume and observed the convergence of results for grid independent solutions. Kuyper and Hoogendoorn [20] investigated laminar natural convection flow in trapezoidal enclosures to study the influence of the inclination angle on the flow and also the dependence of the average Nusselt number on the Rayleigh number. Boussaid et al. [21] studied the thermosolutal heat transfer within trapezoidal cavity heated at the bottom wall and cooled at the inclined top wall. We mention the very interesting studies on natural convection in trapezoidal enclosures filled with either viscous fluid or porous medium by Varol et al. [22,23]. Numerical results indicated that there exist significant changes in the flow and temperature fields as compared with those of a differentially heated square porous cavity. These results lead, in particular, to the prediction of a position of minimum heat transfer across the cavity, which is of interest in the thermal insulation of buildings and other areas of technology. Finally, we mention the paper by Basak et al. [24] studied the heat flow patterns in the presence of natural convection within trapezoidal enclosures with heatlines concept and uniformly and non-uniformly heated bottom wall, insulated top wall and isothermal side walls with an inclination angle. Momentum and energy transfer are characterized by streamfunctions and heatfunctions, respectively, such that streamfunctions and heatfunctions satisfy the dimensionless forms of momentum and energy balance equations, respectively. Finite element method has been used to solve the velocity and thermal fields and the method has also been found robust to obtain the stream function and heat function accurately.

It should be noted also that natural convection within the trapezoidal enclosures filled with Newtonian or non-Newtonian fluids, porous media are highly useful [18–26] for the applications in the greenhouse-type solar stills, desalination, solar collectors, solar cavity receiver, solar distiller, melting and solidification of phase change materials, molten metal processing, solar cooking, designs of buildings and attics, heat recovery or indirect heat exchanges.

2. Basic equations

A sketch of the two-dimensional right-angle trapezoidal cavity filled with a micropolar fluid is presented in Fig. 1 with dimensional Cartesian coordinates $\overline{x}$ and $\overline{y}$. The trapezoidal enclosure is bounded by isothermal cooled vertical wall ($\overline{x} = 0$) of temperature T_c, adiabatic inclined wall ($\overline{x} = L - \overline{y}(L - l)/H$) with a local heater of temperature $T_h(T_h > T_c)$ and adiabatic top and bottom walls. The utilized micropolar fluid is considered to be heat-conducting, isotropic, polar fluid in which deformation of molecules is neglected. Physically, a micropolar fluid includes molecules which can rotate independently of the fluid stream flow and its local vorticity. Therefore micropolar fluid is the medium whose behavior during their flows is affected by the microrotation, i.e. the local rotational motion of fluid molecules contained in a given fluid volume element [27,28]. In this micropolar fluid model, two independent kinematic vector fields are introduced – one representing the translation velocities of fluid particles; and the other representing angular (spin) velocities of the particles, called as microrotation vector [8].

The micropolar fluid flow is supposed to be laminar and the micropolar fluid properties are supposed to be constant except for the density variation which is satisfied to the Boussinesq approximation. Taking into account the theory of Eringen for the micropolar fluid flow the governing equations can be written in dimensional Cartesian

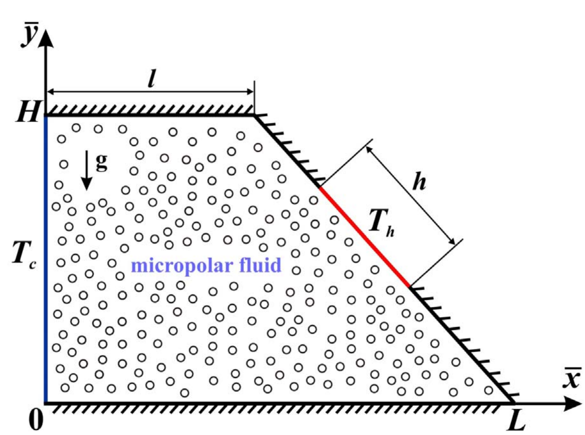

Fig. 1. Physical model and coordinate system.