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Natural vibrations of anisotropic plates with an internal curve with hinges

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ABSTRACT

The objective of this paper is to propose a general algorithm to obtain approximate analytical solutions for the study of the free vibrations of a rectangular anisotropic thin plate with an internal curved line hinge and general restraints. In this system, there exists an intermediate condition that requires the continuity of the transverse displacements. It is well known that the difficulty in choosing admissible functions has been the most significant drawback of the Ritz method. To relax the admissibility requirement the Ritz method, with polynomials as coordinate functions, in conjunction with the Penalty Function method is proposed. This study is focus on different problems related the curved hinge and a natural parametrization is used to treat the mentioned curves. The accuracy of the formulation is ensured by comparing some numerical examples with those available in the literature. Cases not previously treated are particularly analyzed. Frequencies parameters and several sets of vibration mode shapes are included, to provide a better understanding of the dynamical behavior of these plates.

1. Introduction

The plate is probably one of the most common structural elements that have been devised by either scientific or technological interest. It is widely encountered in aerospace, marine, mechanical and civil engineering structures. The dynamical behavior of plates is one of the major concerns in designing this type of structures. It is not the intention to review the literature, consequently only some of the published papers related to the present work will be cited.

Plates with different shapes, boundary conditions and complicating effects have been considered and the frequency parameters were documented in monographs [1,2], standard texts [3–5] and review papers [6,7]. Several complicating effects have been considered such as: elastically restrained boundaries, presence of elastically or rigidly connected masses, variable thickness, anisotropic material, presence of holes, etc. In references [8–11] general studies on vibration of plates with point supports have been presented and vibration of plates with line supports have been developed in references [12–14].

A review of the literature has shown that there is only a limited amount of information for the vibration of plates with line hinges. A line hinge with elastic restrictions in a plate can be used to simulate a through crack along the interior of the plate. Yuan and Dickinson [15] used the Rayleigh-Ritz approach for the study of the free vibration of systems comprised of rectangular plates. The choice of the deflection functions for the component plates were simplified through the use of the concept of artificial springs being introduced at the joints between the plates and at the system boundaries. The necessary continuity and boundary conditions were enforced through allowing the appropriate spring stiffness to become very high compared with the stiffness of the components. Li et al. [16] investigated the vibrational power flow of circular plates with peripheral surface crack modeled as a joint of a local spring. The peripheral surface crack is modeled as a joint of a local spring. The local stiffness of the rotational spring is deduced by using fracture mechanics and strain energy arguments.

Wang et al. [17] studied the buckling and vibration of plates with an internal line hinge by using the Ritz method. Gupta and Reddy [18] studied the exact buckling loads and vibration frequencies of orthotropic rectangular plates with an internal line hinge by employing an analytical method which applies the Levy solution and the domain decomposition technique. Xiang and Reddy [19] developed the first known solution based on the first order shear deformation theory for vibration of rectangular plates with an internal line hinge. The Lévy method and the state-space technique were employed to solve the vibration problem and obtain frequency coefficients values. Huang et al. [20] presented a discrete method to analyze the free vibration problem of thin and moderately thick rectangular plates with an intermediate line hinge. Quintana and Grossi [21] dealt with the study of free transverse vibrations of isotropic rectangular plates with an internal line hinge and elastically restrained boundaries. The problem was solved employing a combination of the Ritz method and the

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Lagrange multiplier method. Grossi [22] used the Hamilton's principle for the derivation of equations of motion and its associated boundary and transition conditions of anisotropic plates with an arbitrarily located internal line hinges with elastics supports and piecewise-smooth boundaries elastically restrained against rotation and translation among other complicating effects. Grossi and Raffo [23] extended the model to analyze anisotropic plates with several arbitrarily located internal lines hinges. Finally, Quintana and Grossi [24] presented a general variational formulation for the free vibrations of laminated thin plates of trapezoidal shape with an internal line hinge. The analysis was carried out by using the kinematics corresponding to the classical laminated plate theory and the eigenvalue problem was obtained by employing a combination of the Ritz method and the Lagrange multipliers method. All of these studies have considered isotropic and anisotropic plates with internal straight lines hinges. However, the literature does not appear to contain any substantial references to analytical models for anisotropic plates with an internal curve with hinges.

According to the statement in the preceding paragraphs the objective of this paper is to propose a general algorithm to obtain approximate analytical solutions for the study of the free vibrations of a rectangular anisotropic thin plate with an internal curve with hinges and general boundary conditions. In this system, there exists an intermediate condition that requires the continuity of the displacements along the line hinge. It is well known that the difficulty in choosing admissible functions has been the most significant drawback of the Ritz method. One way to relax the admissibility requirement may be found by utilizing a constrained optimization technique known as the Lagrangian multiplier method. This method might imply a considerable computational effort. An alternative approach is the Penalty Function Method. The effectiveness of this approach has since been studied by several researchers for various interesting problems and its applicability has also been extended to analyze rigidly connected systems and systems with cracks [25,26]. Ilanko and Monterrubio [27] presented the Rayleigh-Ritz method in an engineering context. The treatment is in a somewhat heuristic form. Nevertheless several textbooks include the corresponding rigorous treatment [28–30].

In this paper a methodology based on a combination of the Ritz method and the Penalty Function Method is used to relax the admissibility requirement on the coordinate functions. Plates with different type of curved hinge are presented and a natural parametrization is used to describe the mentioned curves. To demonstrate the validity and efficiency of the proposed algorithm, results of a convergence study and some particular cases compared with results obtained by other authors are included. Finally, several numerical examples not previously treated are presented.

2. Analysis

Let us consider a rectangular anisotropic plate that in the equilibrium position covers the two-dimensional domain G, with piecewise boundary ∂G elastically restrained against rotation and translation. The plate has one arbitrary intermediate curve with hinges elastically restrained against rotation and translation. A parabola and an inclined line will be considered as particular cases, as it is shown in Fig. 1. In order to analyze the transverse displacements of the system under study we suppose that the vertical position of the plate at any time t, is described by the function $w^{(k)}(x, y, t) \in G^{(k)}$, with $k = 1, 2$, where $x = (x_1, x_2) \in \overline{G}$, $\overline{G} = G \cup \partial G$ and that the domain G is divided by the curve Γ_c into the sub-domains $G^{(1)}$ and $G^{(2)}$, with boundaries $\partial G_i, i = 1, \dots, 4$, where $\partial G_2 = \partial G_2^{(1)} \cup \partial G_2^{(2)}$ and $\partial G_4 = \partial G_4^{(1)} \cup \partial G_4^{(2)}$ (see Fig. 1).

The rectangular plate has a constant thickness h, the rotational and translational rigidities of the elastic restraints along the boundary are respectively given by r_i and t_i, with $i = 1, \dots, 4$. The curve Γ_c has a

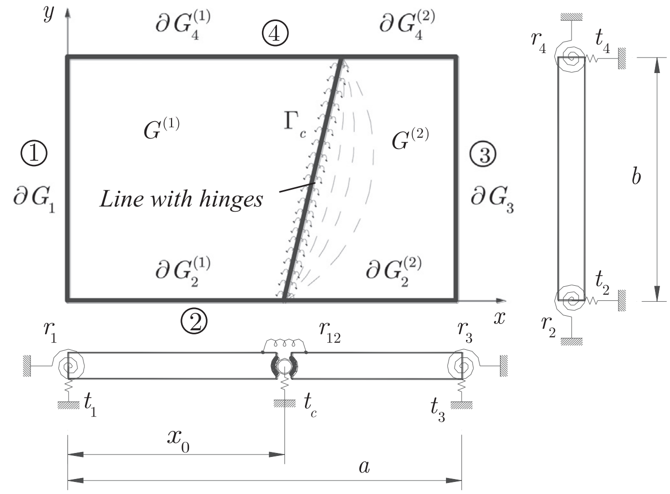

Fig. 1. Mechanical system under study.

rotational elastic restraint r_{12} that is connected to each subdomain of the plate and it also has a translational restraint t_c that one end is connected to the curve with hinges and the other end is connected to a fixed point (see Fig. 1).

The mechanical behavior of Γ_c allows the rotation in the direction of its normal vectors $\vec{n}_c^{(1)}$ and $\vec{n}_c^{(2)}$ when $0 \le r_{12} < \infty$, and the normal displacement of the plate is continuous along Γ_c (see Fig. 2). Therefore, it is allowed a discontinuity in the slope of the displacement at Γ_c.

When the plate makes free vibrations, its displacement is given by a harmonic function of the time, i.e.

$$w^{(k)}(x, y, t) = W^{(k)}(x, y)\cos(\omega t), \quad k = 1, 2, \tag{1}$$

where ω is the radian frequency of the plate and $W^{(k)}(x, y)$ are the plate displacements amplitude of each subdomain.

Considered the established kinematics and basic assumptions of the classical plate theory, the maximum kinetic energy of the vibrating plate is given by

$$T_{\max} = \frac{\rho h \omega^2}{2} \sum_{k=1}^{2} \iint_{G^{(k)}} (W^{(k)}(x, y))^2 dxdy, \tag{2}$$

where ρ is the mass density of the plate.

The maximum potential energy of the mechanical system is given by

$$U_{\max} = U_{P,\max} + U_{R,\max} + U_{T,\max}, \tag{3}$$

where $U_{P,\max}$ is the maximal potential energy of the plate bending, that is given by

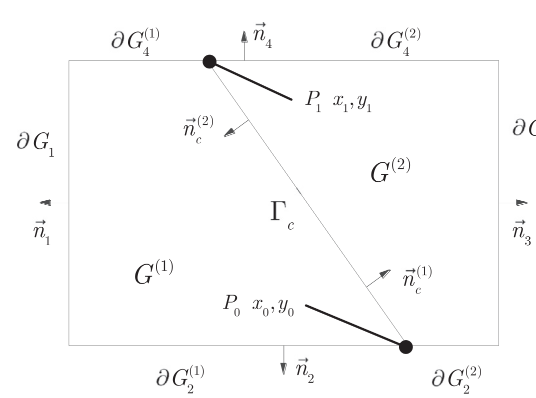

Fig. 2. Mechanical system under study.