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Effect of self-induced electric displacement field on the response of a piezo-bimorph actuator at high electric field

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ABSTRACT

This paper analytically illustrates the response of a piezoelectric bimorph actuator considering the effect of self-induced electric displacement field. In the analysis, we have considered response of the actuator at high electric field. The self-induced electric displacement field exists inside the piezoelectric actuator during its bending. This effect was not considered in the earlier modeling, which typically followed stress-strain based approaches, instead of following the extended Hamilton's principle for electromechanical systems driven by constant voltage source. This electric displacement field affects the tip deflection of a piezoelectric actuator. The new derivation based on Hamilton's principle also shows that a piezo-bimorph's short circuit stiffness, which was considered independent of the piezoelectric coupling coefficient in earlier literature, actually depends on the piezoelectric coupling coefficient. A piezoelectric material of high piezoelectric coupling coefficient can produce a significant self-induced electric displacement field, which can significantly impact the tip deflection and the stiffness of a piezoelectric bimorph actuator. The analytical results are validated with the experimental results, published in earlier literature.

1. Introduction

Piezoelectric bimorphs are widely used as high precision actuators. These actuators can be driven by a low or high electric field depending upon the applications. The electromechanical response of such actuators is almost linear at low electric field. However, the response becomes gradually nonlinear at high electric field excitation. Lee [1] reported a linear constitutive model of piezoelectric laminates for actuation at low electric field. Later, Smith et al. [2] used these linear constitutive equations of piezoelectric elements, and analytically formulated electromechanical response of a piezoelectric bimorph cantilever for low electric field applications. Wang et al. [3] extended the work reported by Smith et al. [2] by introducing an elastic support layer in the middle of a piezoelectric bimorph, and presented an analytical model of such actuators for low electric field applications. Other than these analytical models, several researchers, for instance Detwiler et al. [4], Wang [5] and Vidal et al. [6] addressed finite element model of piezoelectric bimorphs considering linear constitutive equations and low electric field.

Analysis of piezoelectric elements at high electric field is an important problem. Tiersten [7] proposed rotationally invariant second order constitutive equations for the orthorhombic piezoelectric materials assuming small strain and high electric field. Later, Wang et al. [8] and Yao et al. [9] experimentally showed the nonlinear deflection behavior in piezoelectric bimorphs at high electric field. These experimental results motivated the researchers to investigate analytical and finite element models of piezoelectric bimorph actuators for high electric field applications. Considering Tiersten's model [7], Kapuria and Yasin http://10.66.13.70/Digicore/DigiEditPage.aspx?FileName=2331145772608752613239.xml[10] reported finite element modeling of piezoelectric laminates driven by high electric field. Chattaraj and Ganguli [11] considered Tiersten's second order model [7], and analytically illustrated the performance of a rectangular piezoelectric bimorph cantilever actuator for high electric field. However, neither of these studies discussed the effect of self-sensing on the electromechanical response of a piezoelectric bimorph actuator for low and high electric field applications. When a piezoelectric laminate is actuated by an external electric field, the laminate deforms and develops some electric charge internally due to that deformation. Some researches investigated this self-sensing phenomenon, and exploited this internally developed electrical charge for sensing applications, especially in control system designs. Faegh et al. [12] investigated self-sensing in a unimorph piezoelectric microcantilever, and used the device as a biosensor. They incorporated both direct and inverse piezoelectric effect in the same unimorph to establish a capacitance bridge network for detection of ultra small adsorbed masses especially for bio-applications.

Apart from the application of self-sensing as a sensor, it also has an impact on the electroechanical response of a piezoelectric actuator. As

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mentioned earlier, during actuation of a piezoelectric actuator, some electric charge is developed internally due to the deformation. In short circuit condition of a piezoelectric bimorph, the electric field is zero. Whereas, in open circuit condition of a piezoelectric bimorph, the electric displacement field is zero. Therefore, this internally developed electric field or electric displacement field interferes with the external electric field in short circuit or open circuit condition, respectively, and subsequently changes the electromechanical response of the actuator. This electromechanical coupling effectively reduces the deformation of the actuator and it appears that the actuator has been stiffened. Stiffening effect in piezoelectric actuators is discussed in literature by Donadon et al. [13] and Waisman and Abramovich [14], but without adequately analyzing the electrical behavior of piezoelectric bimorph actuators. The existing literature does not adequately address the effect of self-induced electric displacement field on the electromechanical response of a piezoelectric bimorph actuator, which experiences this effect during its bending. The lack in the earlier modeling stems from a derivation approach, which did not use the extended Hamilton's principle for electromechanical system driven by a constant voltage source.

This paper analytically illustrates the effect of self-induced electric displacement field on the electromechanical response of a piezoelectric bimorphm actuator, when it is actuated at high electric field. The self-induced electric displacement field depends on the piezoelectric coupling coefficient κ_{31}. Present analytical formulation shows that the tip deflection, the short circuit stiffness, the capacitance and the block force of a piezoelectric bimorph depend on the self-induced electric displacement field. The analysis also shows that the self-induced electric displacement field significantly affects the tip deflection and the short circuit stiffness of a piezoelectric bimorph for high piezoelectric coupling coefficient κ_{31}. We have verified the present analytical results with earlier published experimental results [8].

2. Background

Electromechanical network problems can be systematically solved by using Hamilton's principle. A variational indicator (VI) is constructed by using the constitutive equations of electromechanical elements to apply the Hamilton's principle for such systems. There are two different forms of Hamilton's principle for solving electromechanical systems depending on the choice of independent variables. First form is based on the *displacement and charge formulation technique*, and the second form is based on the *displacement and flux formulation technique*, which are discussed briefly in the following section [15,16].

2.1. Mechanical and electrical co-energy function

Mechanical potential energy U, which is stored as strain energy inside an elastic structure, is shown in Fig. 1(a). Here, S represents mechanical strain, and σ represents mechanical stress. Mechanical potential energy per unit volume is defined as

$$\frac{U(S)}{\mathbb{V}} = \int_0^S \sigma.\, \mathrm{d}S \tag{1}$$

and mechanical potential co-energy per unit volume is

$$\frac{U^*(\sigma)}{\mathbb{V}} = \int_0^\sigma S.\, \mathrm{d}\sigma \tag{2}$$

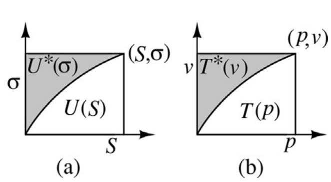

Fig. 1. Mechanical (a) strain energy and constitutive relation between stress σ and strain S, (b) kinetic energy and constitutive relation between momentum p and velocity v.

Here, $\mathbb{V}$ represents volume. Mechanical kinetic energy is shown in Fig. 1(b). Here, p represents momentum, and v represents velocity. Mechanical kinetic energy T is defined as

$$T(p) = \int_0^p v.\, \mathrm{d}p \tag{3}$$

and mechanical kinetic co-energy T^* is defined as

$$T^*(v) = \int_0^v p.\, \mathrm{d}v \tag{4}$$

Electrical energy stored in a capacitor by means of electric field is shown in Fig. 2. Here, Q represents the electric charge, and current $i = \frac{\mathrm{d}Q}{\mathrm{d}t}$. The electric energy U_e stored in a capacitor by means of electric field is

$$U_e(Q) = \int_0^Q V.\, \mathrm{d}Q \tag{5}$$

and, the electric co-energy U_e^* stored in a capacitor is defined as

$$U_e^*(V) = \int_0^V Q.\, \mathrm{d}V \tag{6}$$

The electric energy stored in a capacitor per unit volume is

$$\frac{U_e(D)}{\mathbb{V}} = \int_0^D E.\, \mathrm{d}D \tag{7}$$

and the electric co-energy stored in a capacitor per unit volume is defined as

$$\frac{U_e^*(E)}{\mathbb{V}} = \int_0^E D.\, \mathrm{d}E \tag{8}$$

Here, E represents electric field, D represents electric displacement

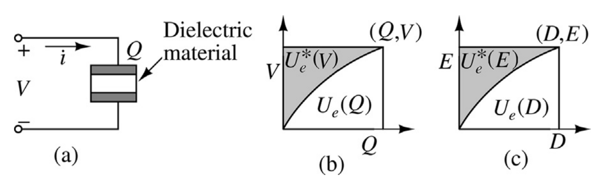

Fig. 2. Capacitor (a) network schematic (b) constitutive relation between voltage V and charge Q (c) constitutive relation between electric field E and electric displacement field D.

field, and $D = \varepsilon E$. Here, ε is dielectric constant. Electrical energy stored in an inductor by means of magnetic field is shown in Fig. 3. Here, λ represents the electric flux, and potential $V = \frac{\mathrm{d}\lambda}{\mathrm{d}t}$. The electric energy U_m stored in an inductor by means of magnetic flux is

$$U_m(\lambda) = \int_0^\lambda i.\, \mathrm{d}\lambda \tag{9}$$

and the electric co-energy U_m^* stored in an inductor by means of

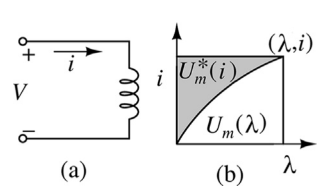

Fig. 3. Inductor (a) network schematic (b) constitutive relation between current i and magnetic flux linkage λ.