International Journal of Mechanical Sciences 121 (2017) 123–142

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

# Trajectory instability and convergence of the curvilinear motion of a hard projectile in deep penetration

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## ARTICLE INFO

*Keywords:*
Deep penetration
Rigid projectile
Trajectory instability
Curvilinear motion
Soil target

## ABSTRACT

This paper studies the trajectory instability and convergence of the curvilinear motion of a hard projectile in deep penetration, based on rigid body dynamics and the differential area force law. It is observed that both sudden (instable) and gradual (curvilinear) changes of the projectile trajectory may happen under certain conditions depending on projectile geometry, target property, impact velocity and other striking parameters. A criterion is introduced to determine the occurrence of trajectory instability. Two characteristic velocities are identified, i.e. (a) the critical impact velocity for the occurrence of trajectory instability, and (b) the characteristic convergent velocity, at which the projectile trajectory turns to a straight line (linear motion) from the curvilinear motion, which is supported by experimental evidence. It is shown that both the critical impact velocity and the characteristic convergent velocity depend linearly on the relative location of projectile's centre of mass. However, the characteristic convergent velocity is independent of the non-axisymmetrical disturbance and impact velocity.

## 1. Introduction

Deep penetration has been studied extensively for various materials, such as metal, concrete and soil media [1–6]. A projectile may hit a target with oblique, pitch and/or yaw angles, as reported in experimental and numerical studies for their effects on penetration process [7–12], which introduced non-axisymmetrical resistance on the surface of the projectile and led to the curved (or curvilinear) projectile trajectory. Early research on projectile trajectory in soil target showed that the projectile trajectory with initial oblique angle may become curvilinear and may even move toward the target surface [13].

In comparison with computational models based on finite or discrete element methods, an analytical model based on the differential area force law (DAFL) requires less material parameters and computational resources for projectile trajectory analysis [13,14]. The DAFL approach was originally proposed by AVCO Corporation in early 1970s. Later, it was adopted by the US Army Waterways Experiment Station (WES) to develop 2-D (PENCO2D) and 3-D (PENCRV3D) codes for projectile trajectory analyses [15,16]. A general framework of non-linear differential equations describing the projectile penetration has been established in previous studies (e.g. [17–21]) based on DAFL approach and semi-empirical resistance function. It has been found that the projectile trajectory may change abruptly around a critical impact velocity, which was considered as a trajectory instability problem [17]. Experimental results in [20] indicated the trajectory convergence, i.e. the projectile trajectory gradually becomes straight after a curvilinear motion in target medium, which has also been observed in [17]. However, this phenomenon was neglected in further studies.

With the increase of penetration capability, the projectile instability becomes an important issue for the accurate prediction of projectile trajectory in deep penetration. In this paper, the curvilinear motion and instability of a hard projectile are studied based on the DAFL method and 3D rigid-body dynamics. The problem statement is given in Section 2. The general model of projectile trajectory under non-axisymmetrical condition is applied to study the curvilinear motion and instability of projectiles in deep penetration in Section 3. The general features of the curvilinear motion and instability of projectile trajectory in soil target are discussed in Section 4, followed by conclusions in Section 5.

## 2. Statement of the problem

When an axisymmetric projectile has a linear motion in a homogeneous target, it is subjected to an axisymmetric axial resistance and its linear motion can be maintained if instability does not occur.

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http://dx.doi.org/10.1016/j.ijmecsci.2016.12.021
Received 15 April 2016; Received in revised form 27 November 2016; Accepted 29 December 2016

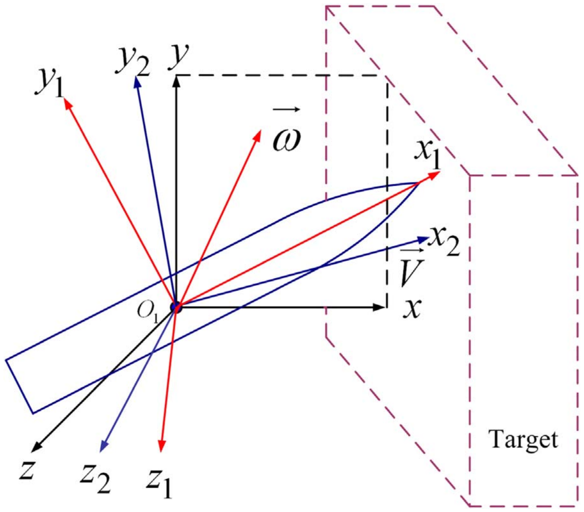

**Fig. 1.** Coordinate frames.

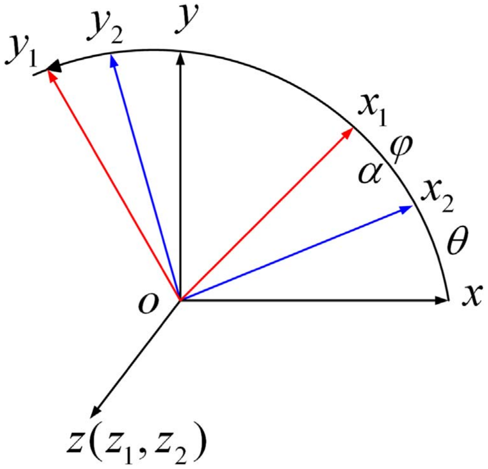

**Fig. 2.** Plane motion of the projectile.

However, an axisymmetric projectile may be subjected to non-axisymmetric resistance due to following three causes, i.e. (i) Initial non-axisymmetric interaction between the projectile and target medium may happen (e.g. the existences of oblique angle, yaw or pitch angle and their angular velocities may cause non-axisymmetric resistant force on the surface of a projectile nose); (ii) Target resistant force on projectile may be non-axisymmetric due to the non-homogeneity of the target medium; (iii) Even the projectile geometry is originally axisymmetric, the projectile may experience non-axisymmetric deformation and mass erosion in early penetration stage, and becomes non-axisymmetric, which introduces non-axisymmetric resistance on the surface of projectile. In this study, we will not consider the non-homogeneity of the target and the deformation and erosion of the projectile.

### 2.1. Coordinate frames of projectile motion

The projectile, target and three coordinate frames are shown in Fig. 1. The target frame is represented by *oxyz* with origin at the projectile mass centre, *xoz* in horizontal plane, axis *ox* opposite to the normal direction of target surface and the axis *oy* in the upright direction. The two other frames $ox_1y_1z_1$ and $ox_2y_2z_2$ in Fig. 1 are the projectile-axis frame and the velocity frame.

In particular, when the normal direction of target, the projectile velocity vector and the projectile-axis are in the same plane as shown in Fig. 2, these three coordinate frames are related by angles $\varphi$, $\theta$ and $\alpha$. The motion of the projectile can be regarded as a plane motion with following relationship

$$\varphi = \theta + \alpha \tag{1}$$

where $\varphi$, $\theta$ and $\alpha$ are termed projectile-axis angle, oblique angle and attacking angle, respectively. Without losing generality, we will consider the plane motion of the projectile in this study. In the rest of paper, these angles with and without subscript ‘0′ denote their respective quantities at the initial (impact) and current moments of penetration.

### 2.2. Initial condition of the projectile

For a homogenous target and an axisymmetric rigid projectile, the non-axisymmetric initial condition is regarded as the key factor for the curvilinear motion and instability of projectile trajectory. The initial condition can be described by three vectors, i.e. the projectile velocity vector, the position vector of the projectile-axis, and the angular velocity vector of the projectile-axis. For the plane motion of a projectile, the initial conditions can also be represented by scalars, e.g. the projectile velocity $v_0$, oblique angle $\theta_0$ and yaw (or attack) angle $\alpha_0$ (or $\varphi_0$) as well as their angular velocities. In a penetration study of an axisymmetric rigid projectile, the initial roll angle and any angular velocities of the projectile are usually insignificant and ignored.

### 2.3. Curvilinear motion and instability of projectile trajectory

Curvilinear motion and instability of projectile trajectory were discussed in [17] as two different problems, in which the trajectory instability was described based on the conventional instability concept by introducing non-axisymmetrical disturbances and imperfections. In a conventional instability problem (e.g. the Euler buckling of an axially-

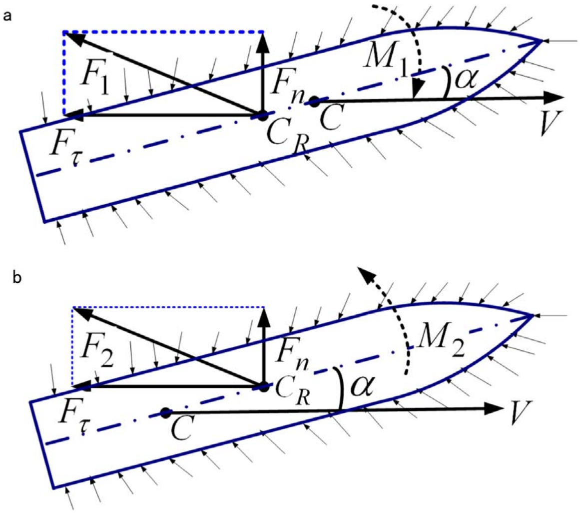

**Fig. 3.** (a) Restoring moment; (b) reverse moment.