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Analysis of the plastic zones of cracks in an elastic-perfectly plastic half-space under contact loading

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A R T I C L E I N F O

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A B S T R A C T

In this paper, a semi-analytic solution is proposed to solve the subsurface stress distribution and plastic zones of an elastic-perfectly plastic half-space with cracks under contact loading. The cracks can be treated as a distribution of edge dislocations with unknown densities based on the distributed dislocation technique. These unknown dislocation densities, contact area and surface pressure distribution can be obtained iteratively when the surface displacement due to the substrate cracks and contact loading is converged by a numerical algorithm according to the conjugate gradient method. The plastic zones at crack tips can be determined by canceling the stress intensity factor (SIF) due to the closure stress and that due to the external applied load based on the Dugdale model of small scale yielding. It is noticed that the plastic zone sizes are affected by the original crack length and depth, yield strength of substrate and loading conditions. This solution might provide guidance for the fracture mechanics analysis of materials with cracks in a half-space.

1. Introduction

Micro-defects, such as cracks and inclusions, commonly exist in engineering materials and structures. These defects are often formed in materials during their manufacturing or utilization process. The presence of these defects can significantly influence the mechanical properties of materials and may eventually result in materials damage and structural failure. Especially, when cracks are beneath the material surface under contact loading, this loading may cause cracks to propagate easily and eventually damage the engineering components, such as gears, bearings, rollers and cams. For example, due to the stress concentration caused by the inclusions in the composite materials under external loading, cracks and dislocations could be initiated and propagated at the vicinities of the inclusions [1].

In the past few years, the analysis of the elastic deformation of materials with cracks or other types of microdefects has been reported in many works [2–9]. For example, the subsurface deformation of materials with cracks under contact loading was investigated by Zhou et al. [10] based on the distributed dislocation technique (DDT) [11]. Then, they also considered the interaction between cracks and inhomogeneous inclusions in a half-space or an infinity space under the external loading on the material properties [12,13]. Afterwards, Dong et al. [14] studied the elastohydrodynamic lubrication effect on the elastic properties of materials with multiple inhomogeneous inclusions and cracks in a half-space. However, all the above-mentioned studies were just concerned the elastic properties of materials with cracks and inhomogeneous inclusions.

In the actual case, crack tips easily yield when the materials are subjected to the external loading since the stresses at crack tips are large even though the applied loading is very small. Therefore, a more accurate description of materials with cracks could be obtained when considering their plastic zones, which is beneficial to analyze the crack initiation and propagation [15–21]. Many factors, such as the loading conditions, material and crack properties, have significant influence on the stress distribution and plastic zones of crack tips. Irwin model [22] and Dagdale model [23] are the earliest theoretical works focused on the plastic zone of crack tips. Irwin model was usually used to estimate the extent ahead of a crack tip. The plastic zone size was determined by setting the crack tip stress to the yield stress. Dugdale model assumed the plastic zone of crack tips was a thin strip and its size could be determined by canceling the SIF due to the applied load and that due to the closure stress.

Recently, numerous works conducted by Yi et al., Hoh et al. and Fan et al. [24–37] were focused on the plastic zone size and crack tip opening displacement of the different types of cracks (Zener-Soroh crack, Griffith crack and arc-shaped crack) based on modified Irwin or Dugdale models. Moreover, Chen et al. [38] also proposed an evaluation method to study the loading conditions on the effect of reverse plastic zone size for a center crack in a plate. All the above mentioned works assumed that cracks were in the infinite solid materials under

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external loading or without loading. However, cracks are usually located near a material surface subjected to contact loading or other loading conditions, which might make the damage effect more prominent compared with the result of cracks in an infinite space under remote stress. This damage analysis needs an accurate study of the plastic zone size of crack tips, surface contact area and pressure distribution, and the subsurface stress field.

This study aims to develop a semi-analytical solution for the stress distribution and plastic zones of an elastic-perfectly plastic half-space with cracks subjected to contact loading. The plastic zone size can be obtained by vanishing the SIF due to the closure stress and that due to the external contact loading. This model can analyze the subsurface stress distribution and the effect of the yield stress of materials, crack length and depth, and loading conditions on the plastic zone size of crack tips, which provides a guideline for the properties of the plastic deformation of materials with cracks.

2. Methodology

2.1. Problem description and solution approach

In this study, a two-dimensional (xOz Cartesian coordinate) contact problem between a rigid cylinder (E_R and υ_R) with the radius R and an elastic-perfectly plastic half-space (E_S and υ_S) with cracks $\Gamma_\varphi(\varphi = 1,2, \cdots m)$, is considered, as shown in Fig. 1(a). The normal load W pushes the cylinder into the half-space to induce the contact problem. The red lines ahead of crack tips are the plastic zones according to the Dugdale model.

In order to solve the governing equation for the substrate stress distribution of this contact problem, cracks can be treated as a distribution of dislocations with unknown densities $\rho^{\perp}$ and $\rho^{\vdash}$ by the DDT. In this way, the original contact problem (Fig. 1(a)) in an inhomogeneous elastic-perfectly plastic half-space can be transformed into a homogeneous contact problem (Fig. 1(b)).

The computational domain with cracks and plastic zones under the contact surface is discretized into $N_x \times N_z$ square elements of the same size $2\Delta_x \times 2\Delta_z$ shown in Fig. 2. Each element is indexed by a sequence of two integers (α, γ) with $0 \le \alpha \le N_x-1, 0 \le \gamma \le N_z-1$. The detailed calculation method to solve the unknown contact area, surface pressure distribution, dislocation densities and surface displacement due to the substrate cracks and contact loading can be found in Appendix A.

The stresses at the sharp crack tips are predicted to be infinite based on the linear elastic analysis. However, they are finite in real materials since the radius of crack tips must be finite. According to the Dugdale model, by assuming that the stresses at the crack tips of the effective crack equal to the yield stress σ_{YS} of matrix material, the stress singularity at crack tips will disappear. To determine the plastic zone size at crack tips, the summing of the SIFs $K_{I\rho}$ due to the closure stress and K_{I} resulting from the applied load must be zero [39]. Hence:

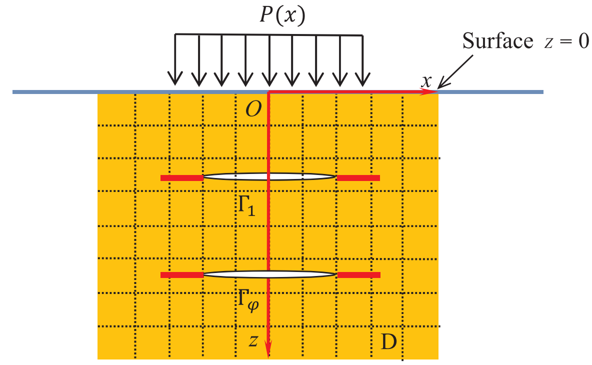

Fig. 2. Discretization of the computational domain into $N_x \times N_Z$ elements of the same size.

$$K_{\mathrm{I}}+K_{I\rho}=0 \tag{1}$$

For the cracks in an elastic-perfectly plastic half-space under contact loading, K_{I} and K_{II} exist simultaneously, then the modified Dugdale model is used in this study to investigate the plastic zone size of crack tips. The condition to solve the plastic zone size of crack tips can be rewritten as follows:

$$\begin{cases} K_I^L+K_{I\rho}^L=0 \\ K_I^R+K_{I\rho}^R=0 \end{cases}, \begin{cases} K_{II}^L+K_{II\rho}^L=0 \\ K_{II}^R+K_{II\rho}^R=0 \end{cases} \tag{2}$$

where the superscripts L and R indicate the left and right crack tips, respectively; the subscripts I and II indicate the Mode I and Mode II cracks, respectively.

The key point for obtaining the plastic zone size of crack tips is to develop the method to calculate the SIFs due to the closure stress and the applied loading. The solution to obtain the expressions of $K_{I\rho}$ and $K_{II\rho}$ is presented in Section 2.2, while the solution to calculate the results of K_{I} and K_{II} is shown in Section 2.3. When the plastic zone sizes are calculated, the subsurface stresses influenced by the plastic zone of crack tips should be updated.

Based on the limitations of the Dugdale crack model (the plastic zone of crack tips must be symmetric), this study just considers the case that the center of crack is same as the loading indentation, which means

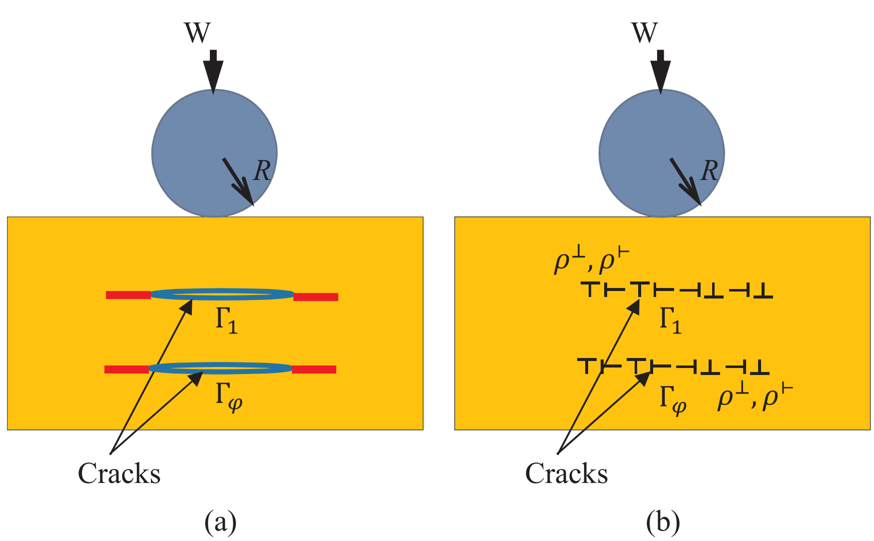

Fig. 1. Schematic of the contact system with the plastic zones (red lines) of the crack: (a) the original problem and (b) the converted homogeneous problem. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)