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# Constitutive boundary conditions and paradoxes in nonlocal elastic nanobeams

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**Abstract**

A debated issue, in applications of Eringen's nonlocal model of elasticity to nanobeams, is the paradox concerning the solution of simple beam problems, such as the cantilever under end-point loading. In the adopted nonlocal model, the bending field is expressed as convolution of elastic curvature with a smoothing kernel. The inversion of the nonlocal elastic law leads to solution of a Fredholm integral equation of the first kind. It is here shown that this problem admits a unique solution or no solution at all, depending on whether the bending field fulfils constitutive boundary conditions or not. Paradoxical results found in solving nonlocal elastostatic problems of simple beams are shown to stem from incompatibility between the constitutive boundary conditions and equilibrium conditions imposed on the bending field. The conclusion is that existence of a solution of nonlocal beam elastostatic problems is an exception, the rule being non-existence for problems of applicative interest. Numerical evaluations reported in literature hide or shadow this conclusion since nodal forces expressing the elastic response are not checked against equilibrium under the prescribed data. The cantilever problem is investigated as case study and analytically solved to exemplify the matter.

*Key words:* Nonlocal elasticity, Integral and differential constitutive laws, Well-posedness, Nanobeams

## 1. Introduction

A challenging paradox of nonlocal mechanics is commonly considered to be faced in looking for the bending solution of elastic beams obeying the elastic integral nonlocal law Eq.(1) according to which the bending field is got by convolution of elastic curvature with the special smoothing kernel of Eq.(3), depicted in Fig.1.

Striking examples are Bernoulli-Euler nonlocal cantilever nanobeams under end-point loading which find applications in microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS) as actuators or sensors.

The paradox, first detected in [1] and later claimed in [2], was that some bending solutions of integral-based nonlocal elastic beams are found to be identical to the classical (local) solution. This affirmation has been repeated several times in literature but a fully clarifying treatment has not yet been contributed.

Recently the issue has been newly drawn to attention by the discussion in [3] where the relation between integral and differential formulations of the nonlocal constitutive law is addressed and a treatment of paradoxical examples is performed by numerical computations based on the integral formulation.

The contradiction between equilibrium and nonlocal constitutive conditions is there considered responsible for preventing the use of the differential constitutive formulation and capable to explain differences between the results obtained by means of the integral formulation with those obtained by the differential formulation.

Our approach is more basic.

It is shown that the nonlocal integral elastic law is equivalent to a problem composed of constitutive differential and boundary conditions. These boundary conditions arise in a natural way in detecting the Green's function of differential problems defined on a bounded domain and provide an effective test to discriminate wether a bending field is obtainable by integral convolution or not.

In Prop.3.1 it will be proven that fulfilment of constitutive boundary conditions by the bending field is necessary and sufficient condition in order to assure existence and uniqueness of the solution of the integral equation defining the corresponding elastic curvature.

At this point a general discussion of the elastostatic problem is appropriate. Firstly we observe that

1. The bending field solution of the elastostatic problem has to fulfil equilibrium with the imposed loading.
2. The elastic curvature has to fulfil kinematic compatibility under the imposed boundary constraints and has to be associated with a bending field that meets the constitutive boundary conditions.

It follows that a solution of the elastostatic problem will exist only if the bending field, univocally detected among the equilibrated ones by imposing the conditions of kinematic compatibility to the corresponding elastic curvature field, will also meet the constitutive boundary conditions. This verification generally fails in cases of applicative interest.

The consequent interpretation of paradoxical examples is different from the one usually adduced in literature.