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Controlling multistability in coupled systems with soft impacts

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ABSTRACT

In this paper we present an influence of discontinuous coupling on the dynamics of multistable systems. Our model consists of two periodically forced oscillators that can interact via soft impacts. The controlling parameters are the distance between the oscillators and the difference in the phase of the harmonic excitation. When the distance is large two systems do not collide and a number of different possible solutions can be observed in both of them. When decreasing of the distance, one can observe some transient impacts and then the systems settle down on non-impacting attractor. It is shown that with the properly chosen distance and difference in the phase of the harmonic excitation, the number of possible solutions of the coupled systems can be reduced. The proposed method is robust and applicable in many different systems.

1. Introduction

The interaction between impacting systems is nowadays extensively investigated. In many systems such as tooling machines, gear boxes, heat exchangers and backlash gear the motion of some elements is limited by a barrier. There are many impact models which give the relations between the interacting system elements. Generally, they can be divided into two groups, i.e., the hard and soft impacts [1,2,4,3]. The hard impacts are modeled by the restitution coefficient [6,7,5]. In this approach the time of contact is infinitely small and the exchange of energy is instantaneous. The second approach (soft impact) assumes the finite, nonzero contact time and a penetration of the base by the colliding body. Hence, the contact is modeled as a linear [8–10], Hertzian [11,12] or other nonlinear [13] spring and viscous damper. The separate equations of motion describe the in-contact and out-of-contact dynamics.

The numerous works have been devoted to phenomena induced by the impacts. The bifurcation scenarios and implication of grazing events are quite well understood [14–17]. There are a few studies which focus on the systems where impacts between coupled oscillators are transient. Blazejczyk-Okolewska et al. [18] show that impacting systems can synchronize (via the exchange of energy during the contact) in anti-phase on chaotic attractor. The impacts can be considered as a discontinuous transient coupling which disappears once the interacting systems reach the synchronous solutions.

The phenomenon of synchronization is commonly encountered in non-linear systems [19–21]. Generally, in coupled mechanical systems one can observe two types of synchronous motion, i.e., the complete and the phase synchronization [22–25]. As the coupling between mechanical oscillators (two directly interacting bodies or via spring, damper or inerter) is always bidirectional; when systems are completely synchronized the value of coupling force is equal to zero, and only if common motion is perturbed the systems once again start to interact (note that for non-mechanical systems it is not always true). This is the straightforward analogy to the above mentioned discontinuous transient coupling via impacts, where the perturbation of stable non-impacting solution leads to the appearance of transient impacting motion (coupling).

In this paper we demonstrate the idea and present solution to reduce complexity via transient impacts. We consider systems of two identical oscillators and assume that interaction between them occurs through soft impacts. When the systems are uncoupled we observe multiple stable attractors for each subsystem. Using a piecewise transient coupling to another identical subsystem we can change the number of stable attractors and, in many cases, specify on which attractor both systems settle.

The paper is organized as follows. In Section 2 we consider a simple model which is used to demonstrate the main idea of our approach and define the notations introduced to describe existing periodic states. In the next section we present and describe the how via discontinuous coupling we can decrease number of solutions in the complex systems with many co-existing periodic solutions of different type. The possible coexistence of impacting and non-impacting solutions is discussed in Section 4. Finally, in Section 5 the conclusions are given.

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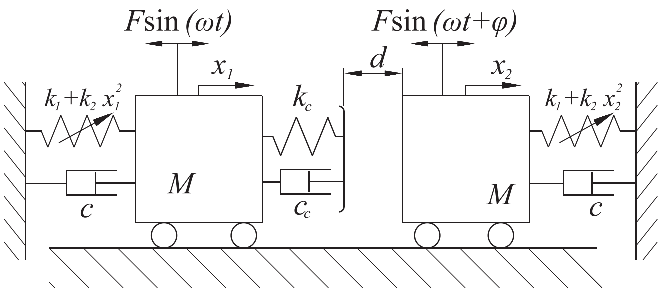

Fig. 1. Model of two discontinuously coupled Duffing oscillators.

2. Duffing systems

In this section we show the main idea of our approach using a simple example. First, the considered model is introduced and the notations used to classify the existing solutions are defined. Then, we present the results of the numerical analysis.

2.1. Model of the system

The system considered in this subsection consists of two identical Duffing oscillators shown in Fig. 1. Oscillators in their steady states (at rest) are separated by the distance d, and the impacts which could occur between them are of the soft type due to the presence of a spring with stiffness k_c and a viscous damper with damping coefficient c_c. Two Duffing oscillators have masses M each and are damped by viscous dampers with damping coefficient c. The spring connecting each oscillator to the wall is nonlinear and of hardening type, where both stiffness coefficients are positive: $k_1 > 0$ and $k_2 > 0$. Both Duffing systems are driven by harmonic forces with the amplitude F and the frequency ω but there is a phase shift between these forces. Forcing of the first oscillator has fixed phase (equal to zero) while for the second one there is a phase shift φ which is used as a control parameter ($\varphi \in \langle 0, \ 2\pi)$).

The whole system is described by the following equations of motion:

$$M\ddot{x}_1 + k_1 x_1 + k_2 x_1^3 + c\dot{x}_1 + F_C = F\sin(\omega t) \tag{1}$$

$$M\ddot{x}_2 + k_1 x_2 + k_2 x_2^3 + c\dot{x}_2 - F_C = F\sin(\omega t + \varphi) \tag{2}$$

where F_C describes the forces generated by the discontinuous coupling and is given by the formula:

$$F_C = \begin{cases} 0 & \text{for } x_1 - x_2 < d \\ k_c((x_1 - x_2) - d) + c_c(\dot{x}_1 - \dot{x}_2) & \text{for } x_1 - x_2 \geq d \end{cases} \tag{3}$$

The values of the parameters are as follows: $M = 1.0$ [kg], $k_1 = 1.0$ [$\frac{\mathrm{N}}{\mathrm{m}}$], $k_2 = 0.01$ [$\frac{\mathrm{N}}{\mathrm{m}^3}$], $c = 0.05$ [$\frac{\mathrm{Ns}}{\mathrm{m}}$], $F = 1.0$ [N], $\omega = 1.3$ [$\frac{1}{\mathrm{s}}$], $k_c = 8.0$ [$\frac{\mathrm{N}}{\mathrm{m}}$], $c_c = 10.0$ [$\frac{\mathrm{N}}{\mathrm{m}}$]. Distance between system d and phase shift in excitation φ are controlling parameters. Introducing dimensionless time $\tau = t\omega_1$, where $\omega_1 = 1$ [$\frac{1}{\mathrm{s}}$], reference length $l_r = 1.0$ [m] and mass $m_r = 1$ [kg] we transform the equations (1)–(3) into dimensionless form in which dimensional parameters are replaced by the following non-dimensional parameters:

$$M' = \frac{M}{m_r}, \quad k'_1 = \frac{k_1 l_r}{m_r \omega_1^2}, \quad k'_2 = \frac{k_2 l_r^2}{m_r \omega_1^2}, \quad c' = \frac{c}{m_r \omega_1},$$
$$F' = \frac{F}{m_r l_r \omega_1^2}, \quad \omega' = \frac{\omega}{\omega_1}, \quad k'_c = \frac{k_c l_r}{m_r \omega_1^2}, \quad c'_c = \frac{c_c}{m_r \omega_1}, \quad d' = \frac{d}{l_r}.$$

We perform transformation to the dimensionless form in the way that enables to hold the values of parameters, hence: $M' = 1.0$, $k'_1 = 1.0$, $k'_2 = 0.01$, $c' = 0.05$, $F' = 1.0$, $\omega' = 1.3$, $k'_c = 8.0$, $c'_c = 10.0$. For simplicity all of the primes used in definitions of dimensionless parameters will be omitted hereafter in the analysis.

2.2. Notations for the periodic solutions

We introduce the notations that enable to describe all periodic states of two impacting oscillators, hence we can classify all solutions that can occur in the considered system. To recall, we assume that the left (first) system is a reference system, hence its phase of excitation and position are fixed while for the right (second) system the phase of excitation φ can vary in the range form 0 to 2π and the oscillator's position can be changed to decrease or increase the distance d. The solution of the left oscillator is described in the following way:

$$\mathrm{L}_{pl}^{nl}$$

where: *nl* is the number of the attractor (in case of multiple attractors of isolated oscillator) and *pl* is the period of given attractor in respect to the period of excitation (we assume that solutions are periodic). Similarly, the solution of the right oscillator is given by:

$$\mathrm{R}_{pr}^{nr}$$

where: *nr* is the number of the attractor (in case of multiple attractors of isolated oscillator), *pr* is the period of given attractor. To define the solution of the interacting oscillators system we will use the following notations:

$$\mathrm{L}_{pl}^{nl}\mathrm{R}_{pr-s}^{nr}.$$

where s is the shift in phase between the systems given by an integer number when the period of solution is longer than the period of excitation i.e, $s = 1$ for 2π shift and so on.

The best example to describe the importance of s is a case when we have two identical systems both with the same period-2 solution (i.e. their response periods are twice longer than the period of excitation). In Poincaré map, for both systems, we observe two dots. Let's assume that the position of the first oscillator, at the sampling moment of time, is in one of the dots. Then, the second oscillator can be either in the same position ($s = 0$) or in the second dot when its phase is shifted by one period of excitation ($s = 1$). Number of possible shifts is equal to the greatest common divisor (gcd) of both systems solutions' periods. Let us now consider an example where both oscillators have period-2 and period-5 co-existing solutions. If the first oscillator is on period-2 solution and the second one is on period-5 solution only one configuration is possible because $\gcd(2, 5) = 1$, so we have one possible value of $s = 0$. In the other case where both oscillators have period-4 and period-6 co-existing solutions, the $\gcd(4, 6) = 2$, hence $s = 0$ or $s = 1$.

Fig. 2 demonstrates these examples. In Fig. 2(a) we show possible configuration for systems with periods 2 and 5. In this case the period of the whole system is equal to 10. The upper row shows the sequence of possible positions of the system with period 2 (1st or 2nd dot on Poincare map), while the lower row presents the possible positions of the system with period 5 (1st to 5th dot on Poincare map). It is easy to

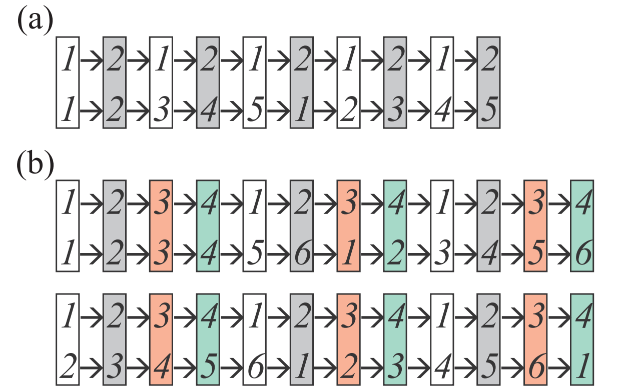

Fig. 2. The possible combinations of the system states for (a) period-2 and period-5 solutions and (b) period-4 and period-6.