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Effect of radial reaction force on the bending of circular plates resting on a ring support

Yong Huang[a], Xian-Fang Li[b,*]

[a] Department of Mathematics, Foshan University, Foshan, Guangdong 528000, PR China
[b] School of Civil Engineering, Central South University, Changsha 410075, PR China

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ABSTRACT

In the classical analysis of bending of a circular plate resting on simple support when subjected to transverse loading, the effect of the radial component of the reaction force is often neglected. This paper analyzes the effect of the radial component of the reaction force on the bending of thin circular plates using the Kirchhoff plate theory. A nonclassical axisymmetric bending problem is studied. By solving the governing equation based on linearization of nonlinear theory of elasticity, two typical cases including uniformly distributed loading over a plate surface and a concentrated force at the plate center are considered. Expressions for the large- and small-scale deflection and rotation are obtained and a non-linear load-deflection relation is given. When neglecting the radial reaction force, a linearized model is reduced and its solution coincides with that of classical thin circular plates. The deflection and load-deflection response curve are graphically presented for centrally-loaded and uniformly-loaded circular plates, respectively. A comparison of the deflections with and without the radial reaction force is made. Obtained results are useful in safety design of linear and non-linear plates under complicated loading.

1. Introduction

As a class of common structural elements, plates are frequently encountered in most engineering structures including aerospace, ocean, mechanical and civil engineering structures. In particular, for special purposes, circular plates are widely used as bridge decks, turbine and flywheel disks, CDs, etc. The simplest theory of plate analysis is the classical Kirchhoff plate theory for thin plates, in which shear deformation and rotational inertia are both neglected. Within the framework of the Kirchhoff plate theory, the static bending of circular plates under the action of transverse loads can be found in many books related to plates [1–3].

The determination of transverse deflection for a plate subjected to applied loading is a fundamental subject. A linear analysis is appropriate if the plate deflection is sufficiently small. For this kind of bending problems, transverse deflections are readily obtainable due to linearity of considered problems. Vivio and Vullo [4] studied axisymmetric bending of circular plates with variable thickness and subjected to symmetrical loading and proposed a new analytical method to determine elastic stresses and deformations with the aid of the solution of the hypergeometric differential equation. Chen and Fang [5] analyzed the deformation and stability of a circular plate under its own weight and supported by a flexible concentric ring and found a stable non-axisymmetric warping of a heavy circular plate. Abbasi et al. [6] formulated a semi-analytical approach to investigate axisymmetric bending of circular plates resting on a Winkler elastic foundation when subjected to a polynomial loading. Some numerical techniques to solve bending and vibration of circular plates have been proposed by finite element method [7], and singular convolution method [8,9], etc. Komaragiri et al. [10] analyzed the mechanical response of freestanding circular elastic films under point and pressure loads based on the simplified Reissner theory.

On the other hand, when the deflections are large, the above solution of linear problems results in serious error. The analysis of large deflections of a plate is necessary [11]. Usually, the von Karman nonlinear plate theory is preferable for large deflections and most studies on large deflections of circular plates mainly focus the von Karman plate theory. For the large deflection problems, it is not feasible for giving an explicit expression for the transverse deflection due to difficulty and complexity of nonlinear problems. Some new techniques, including the perturbation and variation method [2], the incremental load technique [12], iteration technique [13], the shooting method [14], the differential quadrature method [15], have been proposed to solve nonlinear bending problems of circular plates.

* Corresponding author.
E-mail address: xfli@csu.edu.cn (X.-F. Li).

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As we know, the effect of the radial component on transverse bending is often neglected for conventional small deflection analysis. Nevertheless, when deflection is relatively large, the radial component of the reaction force in connection with the corresponding deflection gives rise to bending moment, which in turn affects the deflection. Therefore, consideration of the radial reaction force is necessary. For a horizontally placed simply-supported circular plate, the reaction force at a ring support is always assumed to be vertical in the classical theory. In fact, according to the rule of static mechanics, the reaction force at support position should be orientated in the normal of the surface of the deformed plate after deformation, and it is no longer completely vertical. In other words, it has a horizontally radial component, the effect of which is neglected in the previous studies.

This paper considers the nonclassical bending of circular plates with an emphasis on the effect of radial component of the reaction force at the support rim on the deflection. This paper extends the work on beams in [16] to circular plates. It is organized as follows. In Section 2 we establish the corresponding governing equation of axisymmetric bending of circular plates, where the effect of the radial component of the reaction force at the support rim is included. Section 3 is devoted to bending analysis of two typical cases, a centrally-loaded circular plate and a uniformly-loaded circular plate. Then numerical results of the large- and small-scale deflections and the load-deflection response are given in Section 4. Finally, conclusion are drawn.

2. Governing equations

Consider axisymmetric bending of a circular plate of thickness h under the action of axisymmetric loading, as shown in Fig. 1. The circular plate simply rests on ring circumference with radius a. Cylindrical coordinates r, φ and z are introduced such that the r-axis is chosen as radially outward from the plate center, the φ-axis is chosen along the circumference of the plate, and the z-axis is orientated along the thickness direction and perpendicular to the midplane downward, respectively. Since the problem under consideration is axisymmetric, the variable φ disappears, and only r and z appear in the following analysis.

For axisymmetric problems of elastic materials, the constitutive relations can be expressed as

$$\sigma_r = \frac{E}{1-\nu^2}(\varepsilon_r + \nu\varepsilon_\varphi), \tag{1}$$

$$\sigma_\varphi = \frac{E}{1-\nu^2}(\varepsilon_\varphi + \nu\varepsilon_r), \tag{2}$$

$$\tau_{rz} = G\gamma_{rz}, \tag{3}$$

where $\sigma_r(\varepsilon_r)$ and $\sigma_\varphi(\varepsilon_\varphi)$ are radial and hoop stresses (strains), respectively, $\tau_{rz}(\gamma_{rz})$ is the transverse shear stress (strain), E and G denote Young's modulus and shear modulus, respectively, and ν is Poisson's ratio. In the present study, thin circular plates are only concerned, and it is not difficult to find that the present approach can be extended to treat moderately thick plates. For a circular plate of radius a, adopting the Kirchhoff plate theory we denote the deflection as $w(r)$ being a function of position r, and the radial displacement component can be expressed as

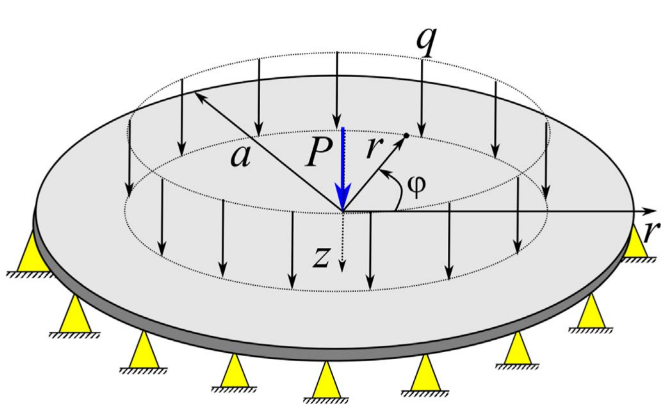

Fig. 1. Schematic of a circular plate resting on simple supports under axisymmetrically distributed loading and concentrated force.

$$u(r, z) = u_0(r) - z\frac{dw}{dr}, \tag{4}$$

where $u_0(r)$ the radial displacement at the midplane due to the radial force.

Using the strains for axisymmetric problems

$$\varepsilon_r(r, z) = \frac{\partial u}{\partial r}, \quad \varepsilon_\varphi = \frac{u}{r}, \tag{5}$$

we can rewrite Eqs. (1)–(3) as

$$\sigma_r = \frac{E}{1-\nu^2}\left(\frac{du_0}{dr} + \frac{\nu}{r}u_0\right) - \frac{zE}{1-\nu^2}\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right), \tag{6}$$

$$\sigma_\varphi = \frac{E}{1-\nu^2}\left(\frac{u_0}{r} + \nu\frac{du_0}{dr}\right) - \frac{zE}{1-\nu^2}\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2w}{dr^2}\right). \tag{7}$$

Define bending moments M_r, M_φ per unit length at radius r by the following integrals

$$M_r = \int_{-h/2}^{h/2} z\sigma_r dz, \quad M_\varphi = \int_{-h/2}^{h/2} z\sigma_\varphi dz. \tag{8}$$

Substituting Eqs. (6) and (7) into the above integrals we get

$$M_r = -D\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right), \tag{9}$$

$$M_\varphi = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2w}{dr^2}\right), \tag{10}$$

where D denotes the bending stiffness, defined by

$$D = \frac{Eh^3}{12(1-\nu^2)}. \tag{11}$$

The radial and hoop forces N_r, N_φ per unit length at r takes

$$N_r = \int_{-h/2}^{h/2} \sigma_r dz = \frac{Eh}{1-\nu^2}\left(\frac{du_0}{dr} + \frac{\nu}{r}u_0\right), \tag{12}$$

$$N_\varphi = \int_{-h/2}^{h/2} \sigma_\varphi dz = \frac{Eh}{1-\nu^2}\left(\frac{u_0}{r} + \nu\frac{du_0}{dr}\right). \tag{13}$$

In the following analysis, the influence of the radial force on the axisymmetric bending will be taken into account. This is due to the fact that once a circular plate is bent, the radial force can cause the bending moment, which in turn gives rise to bending of the circular plate. In this paper, we make an effort to analyze this influence, which is commonly neglected in the classical analysis. For this purpose, subtracting Eq. (10) from Eq. (9), after some algebra one has

$$-\frac{d}{dr}\left(\frac{1}{r}\frac{dw}{dr}\right) = \frac{1}{D(1-\nu)}\frac{M_r - M_\varphi}{r}. \tag{14}$$

In the presence of radial force, based on the incremental deformation theory of elasticity [17] or linearized theory of geometrically nonlinear elasticity [18], the equilibrium equations for axisymmetric bending of circular plates read

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\varphi}{r} + \sigma_r^0\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru)}{\partial r}\right) = 0, \tag{15}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \sigma_r^0\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = 0, \tag{16}$$

where σ_r^0 is initial radial stress ($\sigma_r^0 > 0$ stands for tensile and $\sigma_r^0 < 0$ for