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Incompatible extended layerwise method for laminated composite shells☆

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ABSTRACT

The extended layerwise method (XLWM) was established in the previous studies [1,2] for the laminated composites with multiple delaminations and transverse crack based on the layerwise theory and extended finite element method (XFEM). In order to improve convergence rate, Wilson's incompatible freedom is introduced into the in-plane displacement discretization of the XLWM, and an incompatible extended layerwise method (IXLWM) is established for the laminated composite shells in this paper. The finite formulation of IXLWM is deduced from Hamilton's principle for composite shells with delamination and/or transverse crack. In the numerical examples, the composite beams, plates and spherical shells are used to validate the proposed IXLWM and investigate its convergence rate for the static responses and stress intensive factor (SIF). In addition, the influence of incompatible freedom in different directions on the convergence rate is studied.

1. Introduction

The extended finite element method (XFEM) [3–5] has widely applied to solve the problems that exhibit strong and weak discontinuities in material and geometric behavior, especially for the moving discontinuous problems, as it enables the geometric and materials discontinuity to be independent on the meshing. The applications of XFEM were mainly in elastic crack growths [3,4,6,7], cohesive crack propagation [8], dynamic crack propagation [9–12], bimaterial interface crack [7,13], two-phase flows [14,15] and so on. Recently, the XFEM was applied to simulate the fracture problems of the isotropic and composite plates, such as the delaminations [16–22] and transverse crack [23,24,10,9,25–32]. The XFEM was also applied to simulate delamination and transverse crack growth coupled with the virtual crack closure technique (VCCT) [33,34] or cohesive zone model (CZM) [35–38].

The shell elements method improved by the XFEM were applied to simulate the thick-through cracks or delaminations individually for the laminated composite structures, but there was no work has yet been reported for the typical damage pattern including two kinds of cracks simultaneously. However, the typical damage pattern of composite laminated structures introduced by low-speed impact is a complex three-dimensional crack with layered characteristics. Since it is very difficult to apply XFEM directly to deal with complex three-dimensional crack, the complex three-dimensional crack with layered characteristics can be converted into two two-dimensional crack (delaminations and transverse cracks) by using an appropriate displacements model along thickness direction. In order to simulate the delamination and transverse crack simultaneously, a extended layerwise method (XLWM) was developed for the laminated composite beams, plates and shells by the layerwise method and extended finite element method (XFEM) in our previous studies [1,2]. The XLWM of laminated composites can not only perfectly describe the multiple delaminations together with the thick through or non-thick through transverse cracks, but also accurately calculate the displacement and stress fields of the transverse crack tips and delamination front. Because the XLWM is quasi-3D and the transverse cracks of each single layer are independently described, the distribution of the stress intensity factor (SIF) along the thickness direction can be calculated, and the predicted crack growth angle is different for each mathematic layer. This serves an important advantage compared with the existing shell elements enriched by the XFEM. The XLWM expanded the application of the XFEM in the fracture analysis and prediction of laminated composite structures.

Although the unprecedentedly success of XFEM in approximating the singular field around the crack tip independent of the meshing, it still has been long hindered in engineering applications. As a result, numerous efforts have been made to obtain a improved XFEM. In general, they involve modification on original enrichments [13,39], higher-order enrichments or elements in substitution for original ones [40,41] and combination with subsistent numerical techniques [42,43]. Liu et al. [13] improved the XFEM by enriching the nodes surrounding the crack tip with the first term and higher order terms of the crack tip asymptotic field. Tian and Wen [39] developed an improved XFEM

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(iXFEM) by a globally interpolating approximation based on local least-squares fitting to overcome three difficulties of the existing XFEM: 1) to eliminate the linear dependence and the ill-conditioning issues; 2) to get rid of extra dofs in crack tip enrichment to facilitate optimal mass lumping in dynamic analyses; 3) to be interpolating at enriched nodes to enable direct essential/contact boundary treatments. Iarve [40] modified the XFEM by replacing the step function with a polynomial B-spline approximation functions. The implementation of this method only involves integration of the products of original shape functions and their derivatives without modification of the integration domains. Zamani et al. [41] used higher order terms of the thermo-elastic asymptotic crack tip fields to enrich the approximation space of the temperature and displacement fields in the vicinity of crack tips. The improved accuracy is obtained in this method and the benefit of including such terms is greater for thermal-elastic problems than for either purely elastic or steady state heat transfer problems. Menouillard and Belytschko et. al [42] improved the accuracy of XFEM by using the meshfree approximation as an enrichment in a cluster of nodes about the crack tip. Yu and Liu [43] enhanced the implementation of XFEM for stress analysis around cracks by coupling the generalized finite element method (GFEM) and XFEM. The generalized node are used in a cluster of nodes around the cracks, while the conventional finite element are employed at nodes without cracks, so the cost is reduced and the accuracy of stresses in the vicinity of the cracks is also improved.

The most efforts of the existing improved XFEM concentrate on the enrichment functions and the improvement is limited in local enriched domain. And they do not take the other effects of the crack on the global computational domain into consideration. Actually, in addition to the discontinuity and the singularity of crack, there is also a neglected situation that the occurrence of cracks may lead to additional bending. The performance of isoparametric finite elements employed as basic elements by the XFEM are not well satisfied for bending modes, even though the bending is independent of the cracks. Furthermore, the mode II cracking subjected by the out-plane loads should be not well approximated by the linear element currently used in existing XFEM.

Although our previous works about XLWM had solving some restrictive obstacles of applying XFEM to the typical damage pattern of composite laminated plates, there are two weaknesses need to be conquered in the next investigations. One of the weaknesses is that only the Heaviside function is applied to the delamination in the existing XLWM, it means that the delamination is simulated by the node pairs. The delamination front has to consistent with the element edges, so the delamination region is depended on the finite elements and the front is approximated by the short straight lines (element edges). Furthermore, the general delamination damage region usually detected by the nondestructive evaluation (NDE) in the engineering applications, the extremely complex shape of the crack front should result into the modeling and analysis difficult in the XLWM. The other weakness is that the XFEM employed in XLWM to simulate the transverse crack is traditional methods for the orthotropic materials, for example, the optimal convergence rates does not be guaranteed. In our previous works, the refine mesh is needed for the composite plates with damages, especially the region nearby the delamination front and transverse crack tips.

In 1973, Wilson [44] proposed the incompatible element which introduces extra displacement modes at the element level. Although these extra displacement modes violate inter element compatibility, the accuracy and convergence of these elements in modeling bending modes is effectively improved. In the proposed study, the Wilson's incompatible freedoms are introduced into the in-plane displacement discretization of the XLWM to improve its convergence rate, and an incompatible extended layerwise method (IXLWM) is established for the laminated composite shells with multiple delaminations and/or transverse crack. The remainder of this article is organized as follows: Section 2 presents a briefly introduction of the XLWM for the composite shells, including the displacement fields in the thickness direction, Hamilton principle and finite element formulations. Section 3 presents the mathematic formulations of the IXLWM for the composite shells with multiple delaminations from the Hamilton's principle and the in-plane displacement discretization with Wilson incompatible freedoms, and it is extended to the composite shells with multiple delamination and transverse crack in Section 4. Several numerical examples are carried out in Section 5 for the composite beams, plates and shells. The proposed IXLWM is validated by the XLWM and 3D elastic model. The convergence rate of the proposed IXLWM is compared with that of the XLWM for the static responses and SIF. The influence of the incompatible freedoms in different directions on the convergence rate of IXLWM is investigated. Finally, some remarkable conclusions are drawn in Section 6.

2. A brief review of XLWM

The displacement field of the XLWM in the thickness direction is constructed with the linear Lagrange interpolation functions, the one-dimensional (1D) weak discontinuous function and strong discontinuous function. The strong and weak discontinuous functions are applied in the displacement field along the thickness direction to model the displacement discontinuity induced by the delaminations and strain discontinuity induced by the interface between the layers, respectively. The transverse cracks are simulated in the in-plane displacement discretization based on the XFEM. In order to model the displacement discontinuity of delaminations based on the strong discontinuity functions in the layerwise theory, the nodes of the displacements field along the thickness direction should be located at the top surface, the bottom surface and the middle surface of each single layer. This node strategy is also necessary for the simulation of in-plane transverse cracks. However, the weak discontinuity function is needed in this displacements field to model the strain discontinuity resulted from the interfaces between the layers, and to meet the C_0-continuity of the displacement field.

In the XLWM, the displacements at point $(\xi, \vartheta, \varsigma)$ in the laminated composite shells with multiple delaminations can be expressed as

$$u_\alpha(\xi, \vartheta, \varsigma, t) = \Phi_{\zeta k}(\varsigma) u_{\alpha\zeta k}(\xi, \vartheta, t), \ \ \zeta = i, \ l, \ r \tag{1}$$

where $\alpha = 1, \ 2, \ 3$ denotes the components in the ξ,ϑ and ς directions. $u_{\alpha ik}$, $u_{\alpha lk}$ and $u_{\alpha rk}$ are the nodal freedom, the additional nodal freedom to model displacements discontinuity induced by delaminations and the additional nodal freedom to model strains discontinuity induced by interface between the layers, respectively. k represents the nodes in the thickness direction. The subscripts i, l and r denote the standard nodal freedom, the additional nodal freedom for delaminations and the additional nodal freedom for interfaces, respectively. $\Phi_{ik} = \phi_k(\varsigma)$, and ϕ_k is the linear Lagrange interpolation functions along the thickness direction of the laminated composite shell. $\Phi_{rk} = \Theta_k(\varsigma)$, and $\Theta_k = \phi_k(\varsigma)\chi_k(\varsigma)$ is the weak discontinuity shape function used to model the strains discontinuity in the interface between the layers, where $\chi_k(\varsigma)$ is the one-dimensional signed distance function. $\Phi_{lk} = \Xi_k(\varsigma)$, and $\Xi_k = \phi_k(\varsigma) H_k(\varsigma)$ is the shape function used to model delaminations, where $H_k(\varsigma)$ is the one-dimensional Heaviside function. N is the number of the mathematical layers of the composite shells.

The present layerwise concept is very general in that the number of subdivisions (mathematical layers) can be greater than, equal to or less than the number of the material layers through the thickness direction. A mathematical layer is represented as an equivalent, single and homogeneous layer. If there are continuous and uniform stacking sequences in composite laminated structures, the computational cost of the layerwise theories can be reduced significantly by using the sublaminate concept which makes the number of mathematical layers much less than the number of the material layers. The numbers of the standard freedoms and the additional freedoms for interfaces are $N + 2$