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Deformation and stability of short cylindrical membranes

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Abstract

Analysis of the deformation of cylindrical membranes is important for understanding various mechanical systems, including biological structures such as the incudostapedial joint in the middle ear. Although the deformations of long cylindrical membranes have been extensively investigated, studies of short cylindrical membranes have not been published. Here we present an analytical solution for the deformation of moderate-length and short cylindrical membranes using the Mooney-Rivlin form for the strain-energy function. The membrane profiles and the internal pressures fit well with a finite-element model. We show that both moderate-length and short membranes become unstable when passing a certain limit pressure, and this instability persists even for very short membranes. Furthermore, we show that increasing the initial length, the pre-stretch and the stiffness all tend to stabilize a short membrane. The results may furnish insight into the mechanical behaviour of the incudostapedial joint.

Keywords: Short cylindrical membrane, Analytical solution, Finite-element Method, Deformation, stability

1. Introduction

The ubiquity of thin-walled tubes in technology and nature has inspired many investigations of the deformation and stability of these tubes, including, for example, balloon angioplasty of obstructed arteries or veins [1] and the elasto-capillary collapse of wood fibres [2]. When the tube wall thickness is much smaller than the diameter, the problem may be simplified by neglecting one or more spatial variables and developing an approximate shell theory [e.g., 3] based on the resultant forces and moments across a section and the deformations of the middle surface. If the tube wall is even thinner, further simplification is achieved by considering a membrane approximation and neglecting the bending moments and shear forces [e.g., 4]. (In biology and medicine, the term 'membrane' refers to a thin layer of tissue that covers a surface, lines a cavity, or divides a space or organ. In mechanics, however, 'membrane' refers to a thin structure with a negligible bending stiffness.)

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