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Thermoelastic dissipation of rotating imperfect micro-ring model

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ABSTRACT

Present work considers the investigation of rotating micro-ring gyro resonator model with arbitrary imperfect point masses. Equation of motion of the ring is used to obtain the natural frequency with the effect of thermoelastic damping (TED) induced by the high-frequency vibration. In this regard, the temperature profile through the radial thickness of the model is obtained using the one dimensional heat-conduction equation. And then, the inextensional assumption of the model is employed in terms of displacement fields based on harmonic vibration. In order to express the lower and higher modes of the imperfect model, a sum of density function with respect to angular position is adopted. Also, the rotating speed is discussed including unbalance of the ring and the split of the two modes defined as forward and backward directions. Finally, numerical data for TED effect are obtained as Quality factors (Q-factors), and compared with previous results to verify present work. Moreover, parametric studies are presented for the rotating imperfect micro-ring model.

1. Introduction

In the accurate devices of sensors and actuators, Micro-Electro-Mechanical System (MEMS) is widely used in the advanced technologies. Also, the MEMS have been used in micro- and nano-structures to improve the stability performance of aerospace and industrial fields, etc. Specifically, thermoelastic damping (TED) of the micro-structure is an important factor in the high-frequency vibration model. The damping occurs from the interaction of the structures due to internal friction, and then eliminating and controlling the dissipation effect are more difficult than external damping effect. Thus, the energy dissipation have been considered by numerous workers using Quality factor (Q-factor). Lifshitz and Roukes [1] approximately showed the result of TED effect in the straight beam model using equation of motion. Xi et al. [2] studied Q-factor of the cup-shaped resonator, and applied trimming concept in order to obtain the balance of the structure. Guo and Rogerson [3] presented the effect of thermoelastic coupling on a micro-machined resonator model. Wong et al. [4] extended thermoelastic dissipation theory of a beam to the in-plane flexural vibration of thin rings. Also, Lu et al. [5] investigated thermoelastic vibration and damping of cylindrical shell structures. Prabhakar and Vengallatore [6] analyzed an analytical framework to compute TED in the general class of micro-resonators characterized by structural discontinuities in the form of slots or internal channels. Additionally, Li and Hu [7] suggested a MEMS beam resonator with a general proof mass on a network of suspension beams. Kumar et al. [8] developed a model accounting the contribution of tensile force and thermal conductivity in a nano-resonator. Guo et al. [9] examined the effect of geometry on the TED in micro-beam resonators. Further, Guo [10] explored thermoelastic dissipation of the beam resonators with various aspect ratios. While, Xiao et al. [11] studied optimizing the variable thickness of micro-ring element on the disk resonator gyroscope. Kim and Kim [12] proposed the Q-factor of ring with local stiffness and mass deviations, thus the work shows the variation of the factor according to the imperfections. Pei [13] analytically evaluated rotating micro-disk under thermoelastic coupling with optimizing thickness, inner radius and radial width. Kim et al. [14] presented TED on inextensional vibration of rotating ring, and shows rotational motion makes Q-factor to be higher.

Generally, structural imperfection causes instability resulting from the change of vibration characteristics. Thus, effect of irregularity should be studied for precision and stability of the non-symmetric structure. Fox [15] considered the flexural vibration of circular rings with imperfections represented by small attached masses and springs. Hong and Lee [16] analyzed vibration of circular rings with a small discontinuous deviation and performed experimental study. While, rotating motion is an important factor for structures because the rotation keeps more stable during the motion with the accuracy of the model. Tao et al. [17] investigated the rotating ring vibratory gyroscope based on piezoelectric effect with experimental results. Singh

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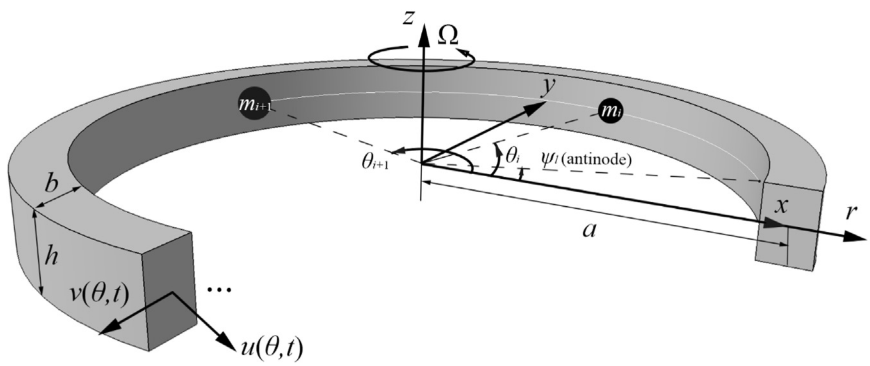

Fig. 1. Coordinate system of ring with imperfections.

and Singh [18] analyzed the governing differential equations of motion for a ring modified to include the effects of rotational speed. Bisegna and Caruso [19] proposed frequency split in imperfect rings by modeling attached masses as Fourier series.

In this paper, rotating imperfect micro-ring model is analyzed, and Q-factor splits due to point masses and rotating are investigated. Additionally, the TED effect is considered based on rotating micro-ring model with point masses using one-dimensional heat conduction equation. Moreover, imperfections are expressed in terms of the amount of mass and angular position. For the validity of present work, results of the Q-factor for the rotating perfect ring model with TED and the isothermal imperfect model are compared with the previous results. Finally, Q-factors of the rotating imperfect micro-ring are presented, and the results are summarized for various parameters.

2. Formulation

Fig. 1 presents the rotating imperfect micro-ring model with arbitrary attached point masses. The global polar coordinate system is used to describe the circumferential strain with local coordinate system. In the figure, a, b and h are mean radius, height, and radial thickness of the model, respectively. Furthermore, Ω [rad/s] is the angular velocity around the z-axis via right-hand rule. As the ring model, isothermal Young's modulus, moment of inertia, and cross-sectional area are defined as E, $I = bh^3/12$ and $A = bh$, respectively. Especially, to represent the imperfections, density with respect to the angular position is expressed as $\rho = \rho(\theta)$.

2.1. Imperfect ring model

In this work, the micro-ring model has dominant mean radius than the radial thickness, then Euler-Bernoulli beam theory can be applied. And only in-plane vibration with respect to z-axis is suitable in this formulation.

Tangential and radial displacements of harmonic motion for the general ring model can be written as

$$u = u(\theta, t) = U_0(\theta)\exp(\mathrm{j}\omega t) \tag{1.a}$$

$$v = v(\theta, t) = V_0(\theta)\exp(\mathrm{j}\omega t) \tag{1.b}$$

in here, $U_0(\theta)$, $V_0(\theta)$, j, ω and t are circumferential and radial displacement component, $\sqrt{-1}$, natural frequency and time, respectively.

As the previous works, the lower mode of vibration for the thin ring is more important, then the deformation of circumferential centerline is small enough, then inextensional assumption is reasonable, thus

$$v = -\frac{\partial u}{\partial \theta} \tag{2}$$

In Eq. (1.a), $U_0(\theta) = \overline{U}_0 \cos(n\theta + n\psi_l)$ is an assumed mode shape of a micro-ring model for n-th mode with attached masses, then Eq. (2) can be applied. Thus, $V_0(\theta) = n\overline{U}_0 \sin(n\theta + n\psi_l)$ is appropriate and then Eq. (1) can be re-written as

$$u = \overline{U}_0 \cos(n\theta + n\psi_l)\exp(\mathrm{j}\omega t) \tag{3.a}$$

$$v = n\overline{U}_0 \sin(n\theta + n\psi_l)\exp(\mathrm{j}\omega t) \tag{3.b}$$

While, the density function of the ring model $\rho(\theta)$ in this study can be represented with the function of i-th imperfect mass m_i and angular position as in Ref. [19]:

$$\rho(\theta) = \rho_0 + \rho_{\mathrm{imp}}(\theta) \tag{4}$$

In here, the final expression of imperfect term $\rho_{\mathrm{imp}}(\theta)$ can be written as

$$\rho_{\mathrm{imp}}(\theta) = \sum_i \frac{m_i}{aA}\delta_i(\theta) \tag{5}$$

where $\delta_i(\theta)$ is Dirac delta function for the location of i-th imperfection at $\theta = \theta_i$.

Further, the imperfection can be re-written using Fourier series as

$$\rho_{\mathrm{imp}}(\theta) = \sum_{k=-\infty}^{\infty}\left[\left(\sum_i \frac{m_i}{2\pi aA}\right)\exp\{\mathrm{j}k(\theta - \theta_i)\}\right] \tag{6}$$

2.2. Thermoelastic damping (TED)

In general, the energy conversion relationship between strain $\varepsilon = \varepsilon_r + \varepsilon_\theta + \varepsilon_z$ and thermal energies yields TED effect. Then, Fourier's heat conduction equation is suitable to obtain the relation. Thus, the 3-dimension heat conduction equation is expressed as

$$\frac{\partial T}{\partial t} - \chi\nabla^2 T = -\frac{E\alpha T_{\mathrm{a}}}{C_{\mathrm{v}}(1-2\mu)}\frac{\partial \varepsilon}{\partial t} \tag{7}$$

where $T = T(r, \theta, z, t) = T_0(r, \theta, z)exp(\mathrm{j}\omega t)$ is the difference of the temperature from the ambient temperature T_a. Furthermore, α, χ, C_{v} and μ are thermal expansion coefficient, thermal diffusivity, heat capacity per unit volume and Poisson's ratio of the material, respectively.

To express TED as displacement and thermal terms in the micro-ring structure, the components of strain with thermal effect can be written as

$$\varepsilon_\theta = \frac{1}{E}\sigma_\theta + \alpha T \tag{8.a}$$

$$\varepsilon_r = \varepsilon_z = -\frac{\mu}{E}\sigma_\theta + \alpha T \tag{8.b}$$

Eq. (8.a) can be re-arranged in terms of ε_θ and T, thus