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Symplectic superposition method for new analytic buckling solutions of rectangular thin plates

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ABSTRACT

Buckling of the rectangular thin plates is a class of problems of fundamental importance in mechanical engineering. Although various theoretical and numerical approaches have been developed, benchmark analytic solutions are still rare due to the mathematical difficulty in solving the complex boundary value problems of the governing high-order partial differential equation. Actually, most available solutions can be categorized as either "accurate" for the plates with two opposite edges simply supported or "approximate" for those without two opposite edges simply supported. In this paper, we present the first work on the symplectic superposition method-based analytic buckling solutions of the rectangular thin plates. A Hamiltonian system-based variational principle via the Lagrangian multiplier method is proposed to formulate the thin plate buckling in the symplectic space. Then the governing equation is analytically solved for some fundamental subproblems which are superposed to yield the final solutions of the original problems. For each problem, a set of equations are produced with respect to the expansion coefficients of the quantities imposed on the plate edges. The existence of the nontrivial solutions of the equations sets the requirement that the determinant of the coefficient matrix be zero, which leads to a transcendental equation with respect to the buckling loads. The buckling mode shapes are obtained by substituting the nontrivial coefficient solutions into the mode shape solutions of the subproblems, followed by superposition. Four types of buckling problems are studied for the plates with combinations of clamped and simply supported edges, without two opposite edges simply supported. The developed method as well as the accurate analytic results is well validated by the finite element method and very few analytic results from the literature.

1. Introduction

The buckling problems of thin plates have attracted continuous attention over the past decades due to their fundamental importance in the field of mechanical engineering. It is well known that many effective numerical methods have been developed for the problems, such as the classic finite element method (FEM), boundary element method, finite difference method, finite strip method, Galerkin method, Rayleigh-Ritz method [1–3], and the modern differential quadrature method [4–7], discrete singular convolution method [8–11], meshless method [12], digital image correlation method [13] and asymptotic method [14]. These methods can normally meet the engineering requirements with acceptable errors; thus they have found vary broad applications in practice. However, accurate analytic solutions are still essential since they could be regarded as the benchmarks for validation of various approximate or numerical methods. Moreover, the accurate analytic solutions are indispensable for some specific issues such as quick prediction of the mechanical behavior and rapid parameter optimization.

In view of the above situation, the present work is aimed to explore the analytic buckling solutions of the rectangular thin plates. In seeking such solutions, one traditionally tries different ways to directly solve the fourth-order partial differential equation (PDE) with respect to the single governing variable, i.e. the deflection of the buckled plate. Any solution approaches within this framework are implemented in the default Euclidean space, and the solutions are often taken in certain forms such as the monomials, polynomials or series, in which the unknown coefficients or constants are determined by letting the solutions satisfy the PDE as well as the associated boundary conditions as much as possible. However, for the rectangular plates without two opposite edges simply supported, it is extremely hard, if not impossible, to find the accurate analytic solutions because the method of separation of variables is invalid for these cases. Nonetheless, continuous efforts are made in recent years on the theoretical analysis of plate buckling, some of which are reviewed here for a better knowledge of the developments in the area. Liew et al. [15] investigated the

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buckling behavior of rectangular Mindlin plates having two parallel edges simply supported by applying the concept of state space to the Lévy-type solution method to obtain the closed-form critical loads from the governing differential equations. A similar approach was adopted by Xiang et al. [16] to study rectangular thin plates with the same boundary conditions subjected to both intermediate and end uniaxial loads. Gorman [17] employed the semi-inverse superposition method to obtain the buckling loads for a family of elastically supported rectangular plates subjected to one-directional uniform in-plane loading. Kang and Leissa [18] formulated an exact solution procedure for the buckling analysis of rectangular plates having two opposite edges simply supported when these edges were subjected to linearly varying normal stresses, where the power series method (i.e., the method of Frobenius) was applied. Jana and Bhaskar [19] developed a rigorous superposition approach for plane stress analysis of the loaded simply supported rectangular plate, and the resulting non-uniform in-plane stress field was fully accounted for in the subsequent stability analysis which was based on the Galerkin's approach with a complete set of admissible functions. A semi-analytical approach based on the variational principal of total energy minimization and the iterative extended Kantorovich method for the buckling analysis of symmetrically laminated rectangular plates with general boundary conditions was presented by Shufrin et al. [20]. Liu and Pavlović [21] proposed an analytical approach for the elastic stability of simply supported rectangular plates under arbitrary external loads through the use of exact solutions for the in-plane stresses and the adoption of double Fourier series for the buckled profiles in the Ritz energy technique. The same approach was later extended by Mijušković et al. [22] to plates with different boundary conditions under arbitrary in-plane compression loads. Ruocco and Fraldi [23] developed an analytic approach for the buckling analysis of rectangular plates under mixed boundary conditions, where the displacement was assumed to be the scalar product of a vector containing four prescribed functions, responsible for the behavior of the plate in one direction, and a vector of four unknown functions depending on the variable in another direction. Thai and Choi [24] obtained the Lévy-type solutions for rectangular plates with two opposite edges simply supported and the other two edges having arbitrary boundary conditions based on two variable refined plate theory. Khorshidi and Fallah [25] applied the Navier method to investigate the buckling of simply supported functionally graded nano-plates based on the exponential shear deformation theory and nonlocal elasticity theory. Li et al. [26] employed a transfer function method to study the initial buckling behavior of thin plates with different boundary conditions resting on tensionless elastic or rigid foundations.

It should be noted that, although various theoretical approaches have been developed, accurate analytic solutions are still rare due to the mathematical difficulty in handling the complex boundary value problems of the governing high-order PDE. An alternative novel solution approach is the symplectic approach [27–32]. It is carried out in the symplectic space rather than in the Euclidean space, where the governing equation is a matrix equation, with a state vector containing multiple variables to be solved. In the symplectic space, some analytic solutions can be achieved by the separation of variables and symplectic eigen expansion. With a proper extension of the symplectic approach, a symplectic superposition method has been developed in recent years to analytically treat both the bending and vibration problems of rectangular plates [33–38]. The method combines the advantages of the symplectic approach and the superposition technique such that the analytic solutions can be obtained in a rational way without predetermining any forms, and the solution procedure is generally applicable to various commonly used boundary conditions such as clamped, free, simply supported and slidingly clamped (also known as "guided") conditions. However, it must be emphasized that the symplectic superposition method for plate buckling problems was not developed.

In the following, we present the first work on the symplectic superposition method-based analytic buckling solutions of the rectangular thin plates. We first formulate the thin plate buckling in the symplectic space by a Hamiltonian system-based variational principle via the Lagrangian multiplier method. Then the obtained governing equation is analytically solved for some fundamental cases which constitute the subproblems to be superposed to yield the final solutions of the original problems. For each problem, a set of equations will be produced with respect to the expansion coefficients of the quantities imposed on the plate edges. The existence of the nontrivial solutions of the equations sets the requirement that the determinant of the coefficient matrix be zero, which leads to a transcendental equation with respect to the buckling loads. The buckling mode shapes are obtained by substituting the nontrivial coefficient solutions into the mode shape solutions of the subproblems, followed by superposition.

For the sake of convenience, we focus on the rectangular plates with combinations of clamped and simply supported edges, and the plates with two opposite edges simply supported are not under consideration because they can be solved analytically using the classical Lévy-type semi-inverse method. Accordingly, the plates with three types of boundary conditions are investigated, i.e. the fully clamped plates (CCCC), plates with one edge simply supported and the other edges clamped (CCCS), and plates with two adjacent edges clamped and the other edges simply supported (CCSS), where C denotes a clamped edge, S a simply supported edge, and the boundary conditions are specified in a clockwise sense. The uniaxial uniform in-plane loads are applied either at one pair of opposite edges or at another pair; thus a CCCS plate can be loaded in two ways, one with the loads applied at the two opposite clamped edges, and another one with the loads applied at the simply supported edge and its opposite clamped edge. Consequently, there are four types of problems to be studied in this paper.

2. Governing equation in the symplectic space for buckling of a thin plate

The Hellinger-Reissner variational principle for buckling of a thin plate within the domain Ω in the rectangular coordinate system (x, y), ignoring the boundary conditions, is [34]

$$\delta\Pi_{H-R} = \delta \iint_{\Omega} \left\{ -M_x \frac{\partial^2 w}{\partial x^2} - 2M_{xy}\frac{\partial^2 w}{\partial x \partial y} - M_y \frac{\partial^2 w}{\partial y^2} - \frac{1}{2D(1-\nu^2)}[M_x^2 + M_y^2 - 2\nu M_x M_y + 2(1+\nu)M_{xy}^2] + \frac{1}{2}\left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right]\right\} \mathrm{d}x\mathrm{d}y = 0, \tag{1}$$

where the functional Π_{H-R} is the generalized potential energy, w is the out-of-plane deflection of the plate, M_x and M_y are the bending moments, M_{xy} is the twisting moment, D is the flexural stiffness, ν is the Poisson's ratio, N_x and N_y are the normal membrane forces, N_{xy} is the shearing membrane force. The variational Eq. (1) yields the equilibrium equations as well as the relations between the internal forces and the deflection, two of which are $M_x = -D(\partial^2 w/\partial x^2 + \nu \partial^2 w/\partial y^2)$ and $M_{xy} = -D(1-\nu)\partial^2 w/\partial x \partial y$. To form the Hamiltonian system-based variational principle, we eliminate M_x and M_{xy} by substituting the above two relations into Eq. (1), and introduce two new quantities, T and θ, via a term $T(\theta - \partial w/\partial y)$, where T is a Lagrangian multiplier. Π_{H-R} is thus reformulated as Π_H:

$$\Pi_H = \iint_{\Omega} \left\{ \frac{D}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{D}{2}\left(\frac{\partial \theta}{\partial y}\right)^2 + D\nu \frac{\partial^2 w}{\partial x^2}\frac{\partial \theta}{\partial y} + D(1-\nu)\left(\frac{\partial \theta}{\partial x}\right)^2 - \frac{D}{2(1-\nu^2)}\left(\frac{M_y}{D} + \frac{\partial \theta}{\partial y} + \nu \frac{\partial^2 w}{\partial x^2}\right)^2 + T\left(\theta - \frac{\partial w}{\partial y}\right) + \frac{1}{2}\left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\theta^2 + 2N_{xy}\left(\frac{\partial w}{\partial x}\right)\theta\right]\right\} \mathrm{d}x\mathrm{d}y. \tag{2}$$

Assuming constant normal membrane forces and zero shearing membrane force for convenience, the new variational equation $\delta\Pi_H = 0$ gives

$$\partial \mathbf{Z}/\partial y = \mathbf{H}\mathbf{Z}, \tag{3}$$

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