International Journal of Mechanical Sciences 118 (2016) 67–76

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Stress analysis of a hollow sphere compressed between two flat platens

X. Chen, C. Li, X.X. Wei *

State Key Laboratory of Mechatronics and Control, School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China

ARTICLE INFO

Article history:
Received 17 March 2016
Received in revised form
5 September 2016
Accepted 15 September 2016
Available online 16 September 2016

Keywords:
Hollow sphere
Compression
Flat platens
Elastic constants
Tensile strength

ABSTRACT

The theoretical basis for determination of tensile strength and elastic constants by compression of hollow spheres between two flat platens is provided in this paper. The method of solution follows the displacement function approach, and three displacement functions are introduced so that the fifteen governing equations can be uncoupled analytically. The Hertz contact between the out surface of the hollow sphere and the flat loading platens is considered, and Fourier-Legendre expansion technique is applied on the Hertz contact stress in order to determine the unknown constants in the general analytical expression for stress components. An analytical solution for the stress and displacement components within a hollow sphere compressed between two flat platens is obtained. Numerical results of the analytical solution show that the stress distributions within hollow spheres are not uniform, two tensile stress concentration zones are usually developed at the inner surface and near the loading area along the loading axis of the hollow spheres within both thick and thin hollow spheres, but the maximum tensile stress is usually developed at the inner surface, except few cases for relatively thick hollow sphere with a very small Poisson ratio. The maximum tensile stress developed along the axis of loading can be used to estimate the tensile strength of hollow spheres. The curves obtained by the analytical solution for the ratio between the vertical and horizontal strains at the equator of the out surface of the hollow sphere, together with those for nominal stress versus the normalized displacement between the two flat loading platens, may provide optional way to determine Poisson's ratio and Young's modulus of the hollow sphere, respectively.

1. Introduction

Benefitting from the fast development of new advanced synthesis techniques, various kinds of hollow microspheres have been synthesized for achieving excellent functions or multi-functions in recent years. For example, SnO_2 hollow microspheres have been found to be an ideal potential material for ethanol sensors [1], A novel magnetic polymeric hollow microspheres are synthesized for selective enrichment and rapid separation of phosphopeptides [2], TiO_2 hollow microspheres can be used as a kind of superior adsorption performance for dye removal [3], TiO_2/C hollow sphere can be used for high performance photocatalysis [4], it was proved that hollow glass microspheres are very useful for hydrogen storage and microwave absorption [5,6], hollow ceramics spheres can be used for heat insulation [7], ZnO hollow microspheres have strong violet emission and enhanced photoelectrochemical response [8], $LiNi_{0.5}Mn_{1.5}O_4$ hollow microspheres have been used for high-voltage Li-ion batteries [9], and hollow silicalite spheres have been made for ethanol/water separation by pervaporation [10].

The hollow microspheres usually accompany with mechanical and volume changes during the process of achieving above advanced functions [11,12]. For example, it has been found that lithium ion battery made up of hollow spheres experience significant volume changes during charging and discharging cycles, stresses associated with these large volume changes have been considered as the cause of cracking and pulverization [13]. Mechanical failure of hollow spheres has been identified as the most critical factor for the functional capacity fade [14,15].

In order to well understand the failure mechanism and optimization design the structure of these advanced functional materials made up of hollow microspheres, it is very useful to study the mechanical behavior of the hollow microspheres, which makes it essential to know the tensile strength and elastic constants (such as Young's modulus and Poisson's ratio) of the materials. The most popular and traditional method in obtaining Young's modulus and Poisson's ratio is uniaxial and triaxial compressions of solid circular cylinders of finite length [16], but this traditional method cannot be used for hollow microspheres, since they are usually grown or synthesized by physical and chemical synthesis methods to form the shape of hollow spheres directly, remolding to cylindrical shape certainly cause the disturbance of the materials, which property may not represent the nature of the original

* Corresponding author.
E-mail addresses: chenxi@bit.edu.cn (X. Chen), cexxwei@bit.edu.cn (X.X. Wei).

http://dx.doi.org/10.1016/j.ijmecsci.2016.09.018
0020-7403/

Nomenclature

R	the radius of the out surface of the hollow sphere
R_0	the radius of the inn surface of the hollow sphere
r_0	the radius of the circular contact area
θ_0	the angle subtended at the center of the hollow sphere by the margin of the contact areas
E	Young's modulus of the hollow sphere
E_p	Young's modulus of the flat loading platens
ν	Poisson's ratio of the hollow sphere
ν_p	Poisson's ratio of the flat loading platens
λ, μ	Lamé's constants of the hollow sphere
$p(\theta)$	contact stress acting over the circular contact surface of the hollow sphere as shown in Fig. 3.
p_0	constant pressure acting on the inner surface of the hollow sphere as shown in Fig. 2.
P	the magnitude of the resultant force acting on the flat loading platens as shown in Fig. 2.
r, θ, φ	three coordinates of the spherical polar coordinate system shown in Fig. 1.
u_r, u_θ, u_ϕ	the displacement component in r, θ and φ directions, respectively
σ_{ij} $(i, j = r, \theta, \varphi)$	the stress components
ε_{ij} $(i, j = r, \theta, \varphi)$	the strain components
Φ , Ψ, Γ	three displacement functions
$P_n(\cos\theta)$	Legendre's polynomial of the first kind
A_n, B_n, C_n, D_n	the unknown coefficients to be determined by the boundary condition

hollow spheres at all. In addition, the experimental results of the uniaxial or triaxial compression of composite cylinders made up of hollow spheres only reveal the overall mechanical properties of the composite materials.

Therefore, we seek the possibility to obtain the tensile strength and elastic constants by compression a hollow microsphere between two flat loading platens. Actually many previous studies have been done on the compression of hollow spheres. In particular, a series of typical tests and numerical analysis have been done on hollow spheres by Yu et al. [17–19]. An approximate solution for a very thin hollow sphere under compression based on the large deflection shell theory is derived [20]. In addition, a kind of resonant ultrasound spectroscopy method was used for measure the elastic constants [21], and a dynamic resonant method was proposed to determine the effective Young's modulus of cellular materials [22].

Wei et al. [23] recently provided a relatively comprehensive review on experimental and theoretical studies on mechanical property of hollow spheres and analyzed the non-uniform stress distributions within hollow spheres under the diametrical point loads. The paper mainly focused on the interaction between two neighbor hollow spheres of foams, and the corresponding interaction point loads are actually modeled as uniform stress acting on two small areas on the out surface of the hollow sphere. Based on fifteen governing equations, which is composed of three equilibrium equations, six equations for Hooke's law, and six equations for the relations between the strain and displacement components, the paper used method of change of variables to uncouple the fifteen governing equations, and obtained an analytical solution for the stress distributions within elastic hollow spheres under point loads. The solution cannot be used to determining the elastic constants because the contact between the out surface and the flat loading platens is not considered, the variation of strains and the relation between the displacement and the external load are not analyzed. Up to now, there is no analytical analysis for determining the tensile strength and elastic constants by compression a hollow microsphere between two flat loading platens.

Therefore, in this paper, based on the same fifteen governing equations as Wei et al. [23], a new displacement function method is used, and three displacement functions are introduced in order to simplify the fifteen governing equations to two uncoupled partial differential equations. The interaction between the out surface of the hollow sphere and the flat loading platens is considered and realistically modeled as Hertz contact stress, which is further expressed in terms of Fourier- Legendre series in order to determine the unknown coefficients in the analytical solutions for hollow spheres. The analytical solution for stress distributions and displacement components for hollow spheres under compression between two loading platens is obtained. The mechanical behavior of the hollow sphere is analyzed according to the analytical solution, and it was found that the tensile strength, Young's modulus and Poisson's ratio can be reasonably determined by the proposed method in this paper.

2. Method of solution

In order to determine the elastic constants by compressing hollow spheres between two loading platens, it is essential to know the relation between the relative displacement between two loading platens and the external applied force P and the ratio between two perpendicular normal strains on the surface of the hollow sphere, which make it necessary to obtain the exact analytical solution for the hollow sphere compressed between two loading platens.

Consider a spherical polar coordinate system (r, θ, φ) with the origin locating at the center of a hollow sphere as shown in Fig. 1, the hollow sphere is compressed between two flat loading platens as shown in Fig. 2. The fifteen governing equations for the analysis of the elastic mechanical behavior include the generalized Hooke's law, the relation between the strain and the displacements components and the equilibrium equations, which are the same as Eqs. (1)–(3) for isotropic spheres under the diametrical point loads by Wei et al. [23]. However, the boundary condition considered by Wei et al. [23] cannot reveal the true contact condition between the flat loading platens and the out surface of the hollow sphere as shown in Fig. 2, because the point loads is simply modeled as uniform stress acting on two small areas on the outer surface of

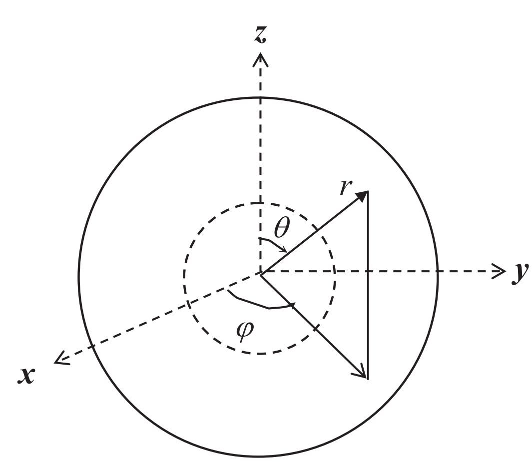

Fig. 1. A hollow sphere in a spherical polar coordinate system.