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Identification of nonlinear hysteretic parameters by enhanced response sensitivity approach

Zhong-Rong Lu, Renzhi Yao, Li Wang *, Jike Liu

Department of Applied Mechanics and Engineering, Sun Yat-sen University, Guangzhou, PR China

A R T I C L E I N F O

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A B S T R A C T

Hysteresis is a ubiquitous phenomenon describing the special nonlinear memory-based relation between the input and the output in many physical systems. Identifying the hysteretic parameters is the first step towards practical application of hysteretic models. In this paper, a general framework for parameter identification of nonlinear hysteretic models is developed based on the enhanced response sensitivity approach. To do so, three typical hysteretic models—Bouc–Wen model, bilinear model with kinematic hardening and bilinear model with equal yielding force are analyzed at first and the general way to model a structure with such hysteretic components is established thereafter. Then, the enhanced response sensitivity approach is presented for inverse parameter identification where the key lies in the sensitivity analysis and the trust-region constraint. Particularly, smoothing procedure is introduced to overcome the non-differentiability of bilinear hysteretic functions for sensitivity analysis of bilinear models. Numerical examples are studied to testify the feasibility and performance of the proposed approach.

1. Introduction

The hysteretic behavior is frequently encountered in many physical systems, such as mechanical systems [1], structural dampers/seismic isolation systems [2,3], friction models [4], oscillatory circuits [1,5] and so on. In definition, hysteresis is a special nonlinear property describing the memory-based relation between the input and the output, that is, the output at a given time instant depends not only on the instantaneous input but also on its past history. Various mathematical models [1,6–9], including Preisach, Ishlinskii, Bouc–Wen, bilinear models, have been developed to describe the hysteretic behavior. Among these models, the Bouc–Wen models are quite general since they can represent a wide variety of softening/hardening, degrading/pinching, smoothly-varying/nearly-bilinear hysteretic behaviors [5,10], but they are often of complex forms; while the bilinear models with kinematic hardening or equal yielding force are the ever simplest hysteretic models and are quite practical for applications in friction modeling and seismic isolation [2] *etc.*, but they suffer from non-smoothness or non-differentiability. In the present paper, the focus is typically on the Bouc–Wen and bilinear hysteretic models and other hysteretic models can be dealt with likewise.

Hysteretic parameter identification or calibration is the first step towards practical application of the nonlinear hysteretic models and it often turns out to be a nontrivial task due to the inherent nonlinearity and memory nature. Two main classes of identification methods have been developed in the literature. In the first class, the identification procedure is transformed into a state estimation problem after discretizing the differential equations into discrete state equations through numerical integrations and treating the parameters as state variables. Then, various techniques are proposed to deal with the state estimation problem and some representative work is: Lin and Corigliano et al. [11,12] proposed to use the extended Kalman filter to identify the state variables, for which, linearization is adopted in order to fit with the general linear Kalman filter framework; Wu and Smyth [10] developed an unscented Kalman filter approach to estimate the parameter by resorting to the unscented transformation; Chang and Shi [13] identified the state variables as well as hysteretic parameters through wavelet multiresolution analysis. The whole procedure for the state estimation seems simple, however, in order to gain good identification, the discrete state equations should well approximate the original differential equations implying that the sampling time step should be as small as possible, and the duration for measurement data should be long enough.

* Corresponding author.
E-mail address: wangli75@mail.sysu.edu.cn (L. Wang).

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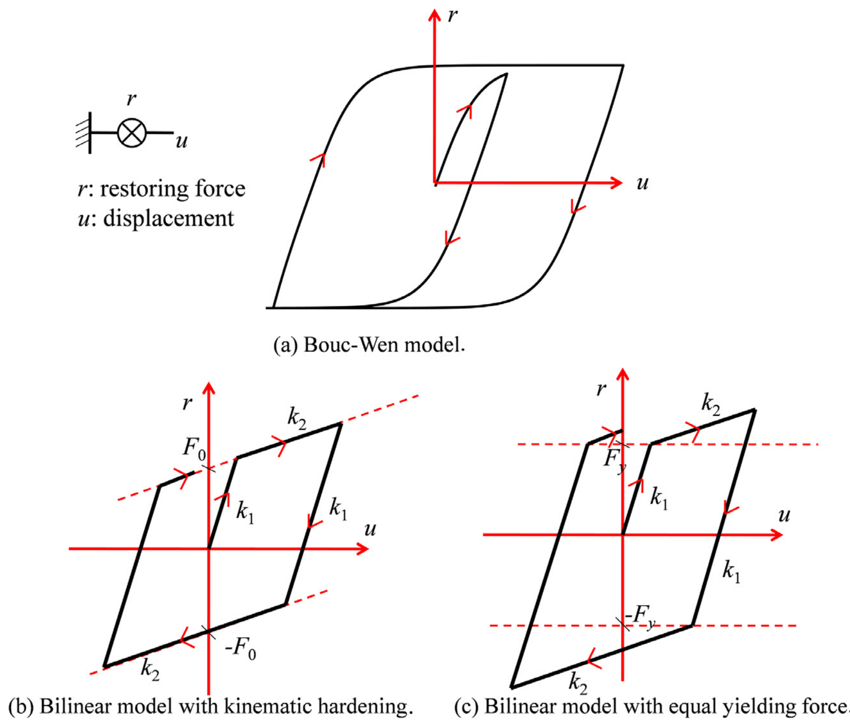

Fig. 1. Nonlinear hysteretic models: (a) Bouc-Wen model, (b) bilinear model with kinematic hardening, (c) bilinear model with equal yielding force.

In the second class, identification of the hysteretic parameters from the measured data is formulated as an inverse problem and is often fulfilled by solving an optimization problem. The objective function is typically defined as the weighted least-squares of the error between the measured data and the derived theoretic data—function of parameters, and then, actual model parameters should minimize the objective function. Different kinds of the measured data have led to different objective functions so well as different identification procedures. Ni et al. [14] proposed to use the frequency-domain displacement data from periodic vibration experiment and then, the Levenberg–Marquardt algorithm was adopted to solve the nonlinear least squares optimization problem. However, getting the experimental frequency-domain data of nonlinear hysteretic systems is not so straightforward as of linear systems and considerable efforts should be taken for large amount of experiment data. In contrast, the time-domain data is much more easily accessed. Yar and Sues et al. [15–18] successfully identified the Bouc–Wen hysteretic parameters by resorting to the time-domain displacement and restoring force data and the multi-stage estimate schemes. The multi-stage therein is mainly invoked to tackle the complex exponential coefficient in the Bouc–Wen hysteretic model. Though satisfactory accuracy as well as convergence was gained, the multi-stage approaches are usually time-consuming.

As is noteworthy, the above two classes of approaches are almost for parameter identification of the Bouc–Wen models and few are devoted to identify bilinear hysteretic parameters. The reason is two-fold. On the one hand, if time-domain displacement and restoring force data of a bilinear hysteretic component is measured, the parameters are explicitly obtainable from the hysteretic loop graph of the bilinear hysteretic model. On the other hand, if other kinds of time-domain data are available, sensitivity analysis is often required for parameter identification process which is possibly infeasible due to the non-smoothness and non-differentiability of the bilinear hysteretic functions. Practically, for structures with bilinear hysteretic components, getting all the displacement and restoring force data may be costly and sometimes impossible; this indicates that new approach is still in demand for practical parameter identification of bilinear hysteretic models.

In the present work, the enhanced response sensitivity approach [19–21], which has been shown successful for structural damage identification, is utilized to establish a general framework for parameter identification of various hysteretic models with arbitrary kind of time-domain measured data. There are several remarkable features of the enhanced response sensitivity approach which are helpful to reach the goal. Firstly, the present work is within the framework of the least squares optimization and therefore, there is no strict constraint on sampling time step. Secondly, the easily available time-domain data is adopted and arbitrarily single kind or combinatory of the data—the acceleration, the velocity, the displacement or even the restoring force can be used to identify the parameters as along as corresponding response sensitivity analysis is performed. Thirdly, the enhanced response sensitivity approach has already been proved to be weakly convergent [20] and this can to some extend guarantee the performance for parameter identification of hysteretic models in this work.

The remainder of this paper is organized as follows. Section 2 introduces three usual hysteretic models and establishes a general model procedure for dynamic motion of structures with hysteretic components. Then, identifying hysteretic parameters is formulated as a nonlinear least-squares optimization problem herein. In Section 3, how to solve the nonlinear optimization problem by the enhanced response sensitivity approach is elaborated. Numerical examples are conducted in Section 4 and final conclusions are drawn in Section 5.

2. Identification problem statement

2.1. Nonlinear hysteretic models

Herein, the Bouc–Wen hysteretic model and two bilinear hysteretic models with kinematic hardening and equal yielding force, respectively are considered (see Fig. 1). Generally, hysteresis describes the memory-based relation between the displacement u and the restoring force r, or mathematically, there is

$$\dot{r} = f(r, u, \dot{u}, \boldsymbol{p}) \tag{1}$$