ARTICLE IN PRESS

International Journal of Non-Linear Mechanics xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Response regimes in equivalent mechanical model of strongly nonlinear liquid sloshing

M. Farid, O.V. Gendelman*

Faculty of Mechanical Engineering, Technion – Israel Institute of Technology, Israel

ARTICLE INFO

Keywords:
Liquid sloshing
Vibro-impact model
Multiple scale analysis, Nonlinear Energy Sink

ABSTRACT

Equivalent mechanical model of liquid sloshing in partially-filled cylindrical vessel is treated in the cases of free oscillations and of horizontal base excitation. The model is designed to cover both regimes of linear and essentially nonlinear sloshing. The latter regime involves hydraulic impacts applied to the walls of the vessel. These hydraulic impacts are commonly simulated with the help of high-power potential and dissipation functions. For analytical treatment, we substitute this traditional approach by consideration of the impacts with velocity-dependent restitution coefficient. The resulting model is similar to recently explored vibro-impact nonlinear energy sink (VI NES) attached to externally forced linear oscillator. This similarity allowed exploration of possible response regimes. Steady-state and chaotic strongly modulated responses are encountered. Besides, we simulated the responses to horizontal excitation with addition of Gaussian white noise, and related them to reduced dynamics of the system on a slow invariant manifold (SIM). All analytical predictions are in good agreement with direct numerical simulations of the initial reduced-order model.

1. Introduction

Cylindrical vessels filled with liquid are used in many fields of engineering for storage of chemicals, gasoline, water, and other possibly hazardous liquids. Oscillations of the liquid in the vessels are referred to as liquid sloshing. Hydraulic impacts caused by the high-amplitude liquid sloshing may have direct and strong harmful effect on the tank stability and robustness [1].

So far, detailed analytical explorations are limited to small-amplitude sloshing in rectangular and cylindrical vessels. While being the most interesting and potentially dangerous, the high-amplitude liquid sloshing in cylindrical tanks still lacks comprehensive analytic description. This problem is notoriously difficult, in particular due to strong nonlinearities caused by the hydraulic jumps and wave collisions with the vessel shells. To get certain (albeit limited) insight into the dynamics of essentially nonlinear sloshing, we adopt and consider an equivalent mechanical model. The latter simulates the effects of hydraulic impacts with the help of high-power smooth potential and damping functions [2–4], following Pilipchuk and Ibrahim [5]. These functions are suitable for numeric simulations, but hardly applicable for analytic treatment. In order to pursue the analytic approach, we further simplify the model and substitute the high-order potential and damping functions with inelastic impact interactions. The resulting model includes both linear components that simulate the linear sloshing responses, and the impact constraints. Traditionally, the impact-induced dissipation is described by Newton's model, which uses a constant restitution coefficient [6,7]. This approximation is not valid for high impact velocities [8], especially for not extremely hard materials, and, of course, not for the fluids. Independent high-power smooth functions for the potential of interaction and the dissipation function somewhat improve the impact approximation. We derive the effective velocity-dependent restitution coefficient from these high-power functions. After this simplification, the model turns out to be treatable analytically, similarly to recently explored dynamics of linear oscillator with attached vibro-impact nonlinear energy sink (VI NES) [7,9,10]. To validate the results, direct numeric simulations with high-power functions are used.

Common reduced-order models present the sloshing liquid as an ensemble of interacting one-dimensional oscillators or pendula. The oscillator models are easier to handle, but fail to represent the vertical liquid motion (e.g. water jets [11]) and vertical excitation, that are better represented by the pendulum model (parametric excitation of liquid-filled vessel modeled by high-exponent potential pendulum by El-Sayad [12], Pilipchuk and Ibrahim [5]). In the same time, the system of one-dimensional oscillators is more common in regulations on the engineering design [13]. Values of basic parameters for both types of models were derived by Dodge [14] and Abramson [15].

Current study deals only with horizontal (for instance, seismic)

* Corresponding author.
E-mail address: ovgend@tx.technion.ac.il (O.V. Gendelman).

http://dx.doi.org/10.1016/j.ijnonlinmec.2017.04.006
Received 30 May 2016; Received in revised form 2 April 2017; Accepted 4 April 2017

Please cite this article as: Farid, M., International Journal of Non-Linear Mechanics (2017), http://dx.doi.org/10.1016/j.ijnonlinmec.2017.04.006

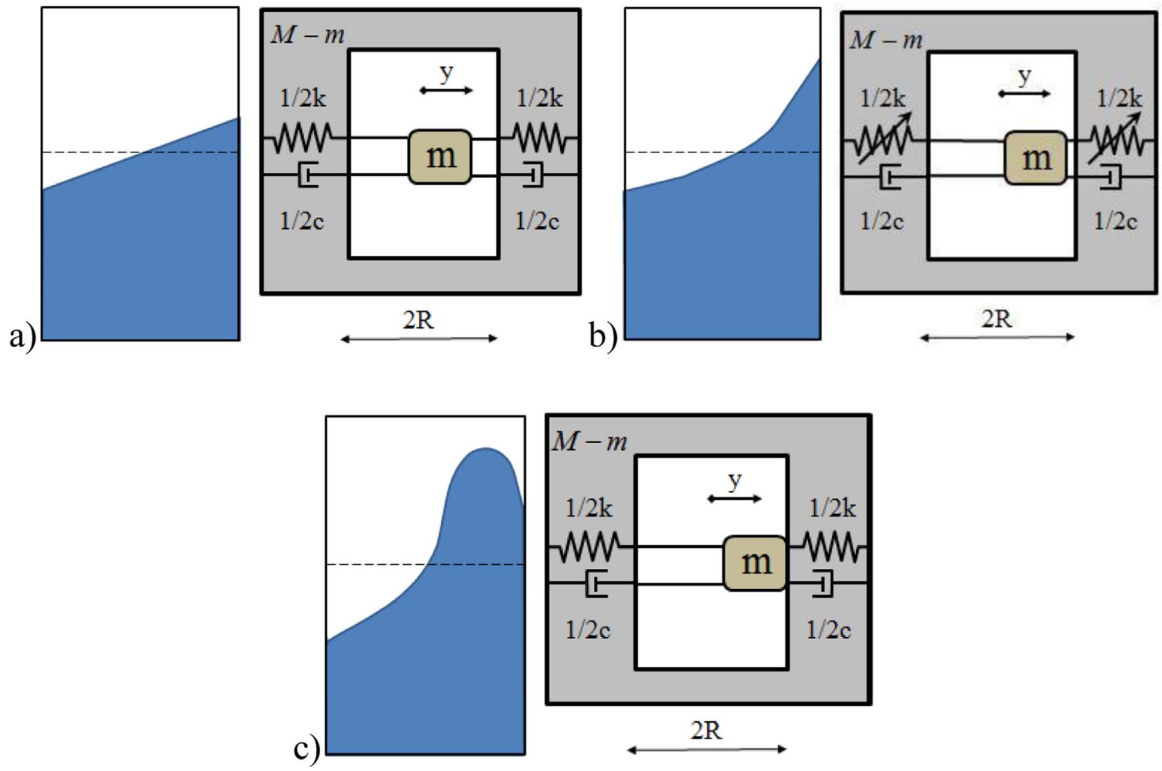

Fig. 1. Sketch of regimes of the free-surface motion of the liquid and their equivalent mechanical models.

excitations. Besides, we take into account only the first sloshing mode, which has the largest modal mass (see [15,18] and Section 4 below). Then, the liquid sloshing in the tank with total mass *M* is approximately described by oscillations of the mass-spring-dashpot system with mass *m*, stiffness *k*, linear viscosity c and displacement y with respect to the vessel centerline. From this simplified point of view, three dynamical regimes can be schematically outlined as presented in Fig. 1.

(a) The free surface of the liquid oscillates around the trivial equilibrium with small amplitude, and remains planar. This regime can be simulated by small oscillations of the linear mass-spring-dashpot system;
(b) The free surface does not remain planar, and oscillates with moderate amplitude. This type of motion can be simulated by adding a weak cubic nonlinearity, and is treated by perturbation methods [5,16,17];
(c) The free liquid surface is urged into a strongly nonlinear motion, accompanied by impacts with the tank walls. This regime can be mimicked by a linear mass-spring-dashpot system with a possibility of impacts.

We use the lumped mass to model both linear liquid sloshing and nonlinear hydraulic impact regimes of the first asymmetric sloshing mode. Motion and bending stiffness of the vessel are also mimicked by the lumped mass and linear stiffness. The vessel mass includes also the non-sloshing portion of the liquid. Dynamical responses in the weakly nonlinear case are considered in previous work [18] and not addressed here. Our main goal is to get insight into the strongly nonlinear (impact) responses of the considered system in the framework of the lumped-mass model.

In Section 2 we describe the model in details, develop the expression for the restitution coefficient as a function of the impact velocity, and formulate the normalized governing equations of motion. In Section 3 an asymptotic approximation for 1:1 internal resonance regime is developed both for the free vibrations and for the case of periodic forcing. In Section 4 numerical validations of the asymptotic results are presented and effect of additional stochastic excitation on the system response is analyzed.

2. Description of the model

2.1. The lumped-mass model

Following the arguments presented in the Introduction, the lumped-mass model comprises the damped linear oscillator (with possible external forcing) and the internal vibro-impact particle with masses *M* and *m*, respectively. Sketch of the system is presented in Fig. 2. Absolute non-dimensional displacements of the primary mass (PM) and the impacting particle (IP) are denoted as $u(t)$ and $v(t)$ respectively. The PM linear stiffness and viscosity are denoted as $\bar{k}$ and $\bar{c}$, respectively.

The IP is located inside a straight cavity in the PM, and in contrast to earlier explored vibro-impact NES [6,7,9], it is attached to it through a linear spring with stiffness k_1. Thus, oscillations of the IP without impacts become possible; they correspond to the linear sloshing regime. The external forcing at the initial stage is considered to be harmonic, with frequency Ω and amplitude $M\mu G$. Without affecting the generality, the cavity length is set to 2.

The modal masses of the asymmetric sloshing modes in cylindrical

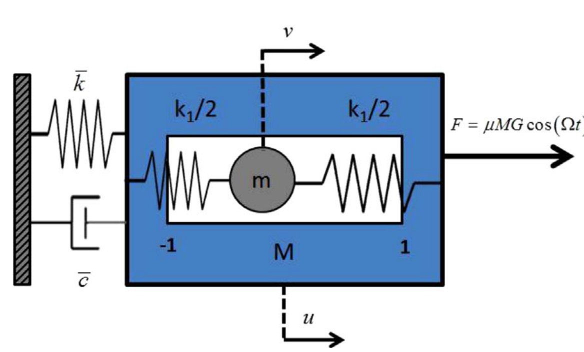

Fig. 2. Sketch of the lumped-mass model.