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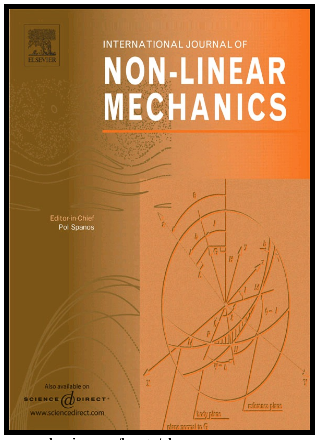

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Periodically Forced Delay Limit Cycle Oscillator

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Abstract

This paper investigates the dynamics of a delay limit cycle oscillator under periodic external forcing. The system exhibits quasiperiodic motion outside of a resonance region where it has periodic motion at the frequency of the forcer for strong enough forcing. By perturbation methods and bifurcation theory, we show that this resonance region is asymmetric in the frequency detuning, and that there are regions where stable periodic and quasiperiodic motions coexist.

1 Introduction

The following equation is perhaps the simplest possible differential-delay equation (DDE) that has interesting dynamics [1],[2] :

$$\frac{dx}{dt}(t) = -x(t-T) - x^3(t) \tag{1}$$

It combines delay with nonlinearity and exhibits a Hopf bifurcation. Although it involves only a first derivative in time, it is, as a DDE, infinite dimensional. Thus unlike first order ODEs, which are dynamically much simpler than second order ODEs (for example autonomous first order ODEs cannot oscillate), eq. (1) supports a limit cycle oscillation.

In this work we shall refer to eq. (1) as a delay limit cycle oscillator (DLCO). Variants of eq. (1) have been the subject of two recent studies. These have involved eq. (1) under linear self-feedback [3], i.e.

$$\frac{dx}{dt}(t) = -x(t-T) - x^3(t) + \alpha x(t) \tag{2}$$

and under a periodic variation within the delay term [4], i.e.

$$\frac{dx}{dt}(t) = -x(t-T) - \epsilon x^3(t), \quad T = \frac{\pi}{2} + \epsilon k + \epsilon \cos \omega t \tag{3}$$

In the present work we continue our investigation of variants of the DLCO by studying the effect of a periodic forcing term:

$$\frac{dx}{dt}(t) = -x(t-T) - \epsilon x^3(t) + \epsilon\alpha \cos \omega t \tag{4}$$