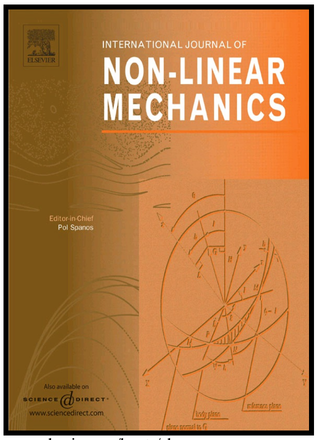

Random Perturbations of Nonlinear Oscillators: Homogenization and Large Deviations

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Random Perturbations of Nonlinear Oscillators: Homogenization and Large Deviations[☆]

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Abstract

This paper considers two prototypical strongly nonlinear oscillators with *weak* dissipation and noise: a pendulum which is acted on by a small noisy torque and opposed by a friction; and the noisy Duffing-van der Pol oscillator. But over a long time the weak dissipative and noise effects can be significant. This paper develops asymptotic techniques based on averaging and large deviations to study the effects of weak noise on the escape from the domain of attraction of stable equilibrium points and limit cycles in phase-space.

Keywords: large deviations, homogenization, Duffing-van der Pol oscillator, energy harvester, SMIB power system

2010 MSC: 37H10, 60F10, 34C29

1. Introduction

In this paper we study strongly nonlinear oscillators with weak dissipation and noise using the techniques of homogenization and large deviations. We study these equations as random perturbations of two-dimensional Hamiltonian systems. The primary goal is to show that the behavior of these strongly nonlinear oscillators can be adequately described by the (slow) evolution of the Hamiltonian, for which simple analytical results can be obtained, and then apply these results to study the transient stability margin of power system with stochastic loads and amplification factors of vibration energy harvesters.

The noisy *Duffing-van der Pol* type equation has been the object of much study in the theory of stability and bifurcations for random dynamical systems. See for example Arnold *et al.* [1] and Namachchivaya *et al.* [2]. In this paper we will restrict our attention to the system with additive white noise. To be specific we consider the stochastic Duffing-van der Pol equation:

$$\ddot{q}_t = \alpha q_t + \beta \dot{q}_t + a q_t^3 + \mathfrak{d} q_t^2 \dot{q}_t + \nu \xi_t, \tag{1}$$

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