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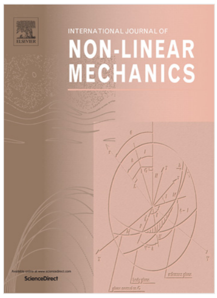

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Dissipative Waves in Real Gases

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Abstract

In this paper, we characterize a class of solutions to the unsteady 2-dimensional flow of a van der Waals fluid involving shock waves, and derive an asymptotic amplitude equation exhibiting quadratic and cubic nonlinearities including dissipation and diffraction. We exploit the theory of nonclassical symmetry reduction to obtain some exact solutions in the limit of vanishing dissipation. Because of the nonlinearities present in the evolution equation, one expects that the wave profile will eventually encounter distortion and steepening which in the limit of vanishing dissipation culminates into a shock wave; and once shock is formed, it will propagate by separating the portions of the continuous region. Here we have shown how the real gas effects, which manifest themselves through the van der Waals parameters $\tilde{a}$ and $\tilde{b}$ influence the wave characteristics, namely the shape, strength, and decay behavior of shocks.

Keywords: *Dissipative waves, Z-K equations, Lie group transformation, Nonclassical symmetry, van der Waals gas, Shock conditions.*

1 Introduction

It is well known that shock waves appearing in a wide range of physical systems are compressive in the sense that these are formed from characteristics approaching the discontinuity from both sides; examples include shock waves in perfect gases and longitudinal waves in solids. Studies have shown that in certain physical systems such as fluids with high specific heats, where the sign of the parameter Γ- so called fundamental derivative, defined as $\Gamma(\rho, \mathrm{s}) = (c\rho^{-1} + c_{,\rho})$, where comma followed by a subscript denotes a partial derivative with respect to the indicated variable, dictates the existence of sonic expansion shocks (see [1, 2, 3]). Here and thereafter ρ, s, and c denote, respectively, the density, entropy, and the speed of sound defined as $c = \sqrt{(p_{,\rho})_{\mathrm{S}}}$ with p as the gas pressure. Here, we examine weakly nonlinear dissipative waves in the unsteady two-dimensional Navier-stokes equations governing the flow of a van der Waals fluid in which the fundamental derivative of gasdynamics changes sign in the vicinity of $\Gamma = 0$. In case of a perfect gas, Γ is always positive and is $O(1)$, but for a van der Waals fluid, Γ may change sign in the pressure density plane and is $O(\epsilon)$ in contrast with the perfect gas case; here ϵ is a measure of the wave amplitude with $0 < \epsilon << 1$. Thus, in the neighborhood of $\Gamma = 0$, for perceptible nonlinear effects, we need time scales longer by an order of magnitude, necessitating the use of fast variables of a higher order magnitude to describe the propagation of signals with perturbed strength $O(\epsilon)$. We then use the method of multiple scales to derive the evolution equation governing the propagation of finite amplitude, two-dimensional, weakly nonlinear dissipative waves in the spirit closer to [4, 5] and use the method of nonclassical symmetries [6, 7, 8, 9, 10] to obtain some exact solutions in the limit of vanishing dissipation. The method of nonclassical symmetries is an extension of Lie's classical method in the sense that it may yield more solutions than those obtained using the classical method. Using this approach, we have found some new exact solutions involving shocks.