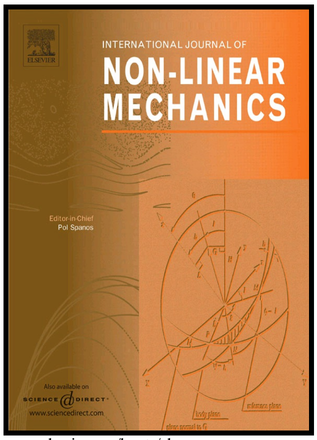
Author's Accepted Manuscript



www.elsevier.com/locate/nlm

Analytical solution in parametric form for the two-dimensional liquid jet of a power-law fluid

A.B. Magan, D.P. Mason, F.M. Mahomed

PII: S0020-7462(17)30294-9
DOI: http://dx.doi.org/10.1016/j.ijnonlinmec.2017.05.001
Reference: NLM2841

To appear in: *International Journal of Non-Linear Mechanics*

Received date: 17 April 2017
Revised date: 28 April 2017
Accepted date: 4 May 2017

Analytical solution in parametric form for the two-dimensional liquid jet of a power-law fluid

A.B. Magan*, D.P. Mason, F.M. Mahomed

School of Computer Science and Applied Mathematics, DST-NRF Centre of Excellence in Mathematical and Statistical Sciences, University of the Witwatersrand, Johannesburg, Wits 2050, South Africa

Abstract

The two-dimensional liquid jet of a power-law fluid is considered. The problem is formulated in terms of the components of fluid velocity which satisfy the continuity equation and the momentum boundary layer equation for a power-law fluid. The multiplier method is used to investigate the conservation laws for the system of partial differential equations and a conserved vector and the corresponding conserved quantity for the two-dimensional liquid jet is derived. The Lie point symmetries of the system of partial differential equations are calculated. A linear combination of the Lie point symmetries is associated with the conserved vector for the liquid jet to obtain the associated Lie point symmetry which is used to generate the invariant solution. An analytical solution in parametric form for the liquid jet is derived. It is found that a solution for the liquid jet exists only for $1/2 < n < \infty$ where n is the power law exponent. The profile of the free surface and the thickness of the liquid jet are compared for a shear thinning fluid with $1/2 < n < 1$, a Newtonian fluid with $n = 1$ and a shear thickening fluid with $1 < n < \infty$.

Keywords: Two-dimensional liquid jet, Power-law fluid, Conservation laws, Conserved quantity, Lie point symmetry, Invariant solution

1. Introduction

In this paper we investigate the analytical solution of a two-dimensional liquid jet of a power-law fluid. A liquid jet is formed when a two-dimensional jet of liquid strikes a plane boundary at right angles and spreads over its surface. A liquid jet has a free surface with a kinematic condition and a stress boundary condition. It is different from a two-dimensional wall jet. A wall jet is formed, for example, when a sluice gate separating two sections of a canal is slightly raised and water flows into the section with the lower water level [1]. The velocity along the jet tends to zero at the outer edge of the wall jet.

Non-Newtonian fluids behave differently from Newtonian fluids because of the nonlinear relationship between the shear stress and strain-rate. A commonly studied non-Newtonian fluid is the power-law fluid. Power-law fluids are sometimes described as generalised Newtonian fluids because the stress is prescribed as a function of a power of the shear rate. They are classified as shear thinning, Newtonian and shear thickening fluids. Due to the relative simplicity of the power-law model, it is used in many engineering applications such as the extraction of crude oil from petroleum products.

The governing equations for shear thinning fluids were first derived by Schowalter [2], and later solved numerically for both shear thinning and shear thickening fluids by Acrivos [3]. Denier and Dabrowski [4] obtained a similarity solution for the boundary layer equations of a power-law fluid.

Schlichting [5, 6] was the first to apply boundary layer theory to jet flows. He considered the two-dimensional free jet of a Newtonian fluid and solved the problem numerically. Bickely [7] obtained an

*Corresponding author
Email address: avnishmagan@gmail.com (A.B. Magan)