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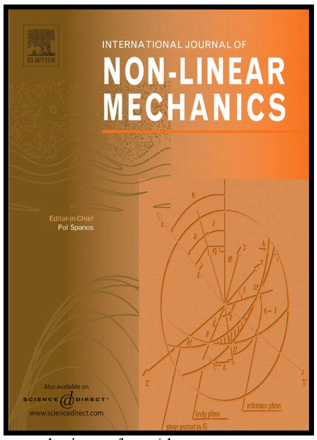


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PII: S0020-7462(17)30046-X
DOI: http://dx.doi.org/10.1016/j.ijnonlinmec.2017.04.010
Reference: NLM2830

To appear in: *International Journal of Non-Linear Mechanics*

Received date: 20 January 2017
Revised date: 19 March 2017
Accepted date: 5 April 2017

Cite this article as: Rami Ahmad El-Nabulsi, Nonlocal-In-Time Kinetic Energy in Nonconservative Fractional Systems, Disordered Dynamics, Jerk and Snap and Oscillatory Motions in the Rotating Fluid Tube, *International Journal o Non-Linear Mechanics,* http://dx.doi.org/10.1016/j.ijnonlinmec.2017.04.010

# Nonlocal-In-Time Kinetic Energy in Nonconservative Fractional Systems, Disordered Dynamics, Jerk and Snap and Oscillatory Motions in the Rotating Fluid Tube

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## Abstract

In this article we generalize the basic theoretical properties of nonlocal-in-time kinetic energy approach introduced in the framework of nonlocal classical Newtonian mechanics for the case of fractional dynamical systems explored in the context of the fractional actionlike variational approach. Two independent fractionally Lagrangians weights are considered independently: the Riemann-Liouville fractional weight and the extended exponentially fractional weight. For each weight, the corresponding nonlocal fractional Newton's law of motion is derived. Three main physical applications were discussed in details: free particles, oscillators and dynamics of particles in a rotating tube with earth frame. A number of differential equations depending on fractional and nonlocal-in-time parameters were obtained and their solutions are discussed accordingly. For specific parameters and particular initial conditions, it was observed that the dynamics exhibit a kind of strange phase plot trajectories that indicate the presence of disordered motions. However one of the main results concerns the physics of particles in the rotating tube which display, for specific values of fractional and nonlocal-in-time parameters, oscillatory motions controlled by the nonlocal-in-time parameter.

**Keywords**: Nonlocal-in-time kinetic energy; fractional actionlike variational approach; nonlocal fractional Newton's law; oscillatory motions of particles in a rotating tube

**AMS Subject Classification (2010)**: 37N05; 26A33; 49S05



## 1. INTRODUCTION

Nonlocal-in-time (NLT) kinetic energy was introduced by Feynman since 1948 for quantum mechanical reasons by shifting the particle position backward and forward in time [**20**]. This approach is motivated in fact from Feynman's observability of the kinetic energy functional which must be written in terms of backward and forward coordinates since position differences should be shifted with respect to each other in time. Feynman's old idea has been used after around 61 years by Suykens by extending local Newtonian mechanics to its nonlocal counterpart and obtaining a generalized classical Newton's law of motion capable of explaining a number of quantum aspects [**47**]. Suykens's idea was extended by Li *et al* in [**29**] for the case of nonconservative systems and a generalization of Newton's second law of motion was derived accordingly. It is noteworthy that backward-forward shifting coordinates (BFSC) were discussed in literature through different arguments, e.g. stochastic quantum mechanics introduced by Nelson in [**38**] who aims to derive the Schrödinger equation from Newtonian mechanics; complexified Lagrangian mechanics [**13**], scale relativity which aims to combine fractal spacetime with relativity principles [**39**], nonconservative quantum mechanics [**52**], local fractional derivative of non-differentiable functions [**9**], fractional actionlike variational approach [**13,14**] among others. It is noteworthy that the term nonlocal-in-time kinetic energy was first coined in Suykens [**47**]. It was inspired on Feynman [**20**] but Feynman was in fact discussing a discrete time numerical