International Journal of Non–Linear Mechanics 92 (2017) 1–7

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Modeling and computation of nonlinear rotating polymeric jets during forcespinning process

Daniel N. Riahi

School of Mathematical & Statistical Sciences, University of Texas Rio Grande Valley (UTRGV) (Brownsville Campus), One West University Boulevard, Brownsville, TX 78520-4933, USA

ARTICLE INFO

Keywords:
Jet flow
Nonlinear jet
Jet stability
Forcespinning
Rotating jet
Polymeric fluid

ABSTRACT

In the usual forcespinning (FS) process, a meso-scale fluid jet is forced through an orifice of a rotating spinneret, where the ambient fluid is air. This leads to the formation of a jet with a curved centerline. In this study we make use of a phenomenological viscosity model for polymeric fluid to investigate the properties of nonlinear polymeric fiber jets during FS process. We apply multi-scale and perturbation techniques to determine the governing modeling systems for such nonlinear rotating jets and their stabilities. First, we calculate numerically the expressions for the leading order nonlinear steady solutions for the jet quantities such as radius, speed, stretching rate, strain rate and trajectory versus arc length, and we determine, in particular, these quantities for different values of the parameters that represent effects due to rotation, viscosity and relaxation time. Next, we calculate the stability of the nonlinear jet versus different types of perturbations. We find that the nonlinear fiber jet flow can be stable in most cases and uncover conditions for which fiber radius reduces and the jet speed or stretching rate increases.

1. Introduction

In the last several years there have been mostly experimental investigations in developing and using the forcespinning method [1,10–12,15,19] which indicated possibility for prediction of nanofibers subjected to some trial-error based cases by FS procedure. FS is a technology that uses centrifugal force due to the externally imposed rotational constraint to produce fibers of micro and nano sizes from different types of melt and solutions, and it allows for the production of nanofibers from a number of materials with important technological applications. An important advantage of FS technique is in its high production rate as well as its applicability to different types of materials such as polymer, composite, metal, ceramic, etc. For example, its production rate per unit time is about over several hundred times more than the corresponding one by electrospinning process [15].

In the last decade or so there have been a number of theoretical and computational modeling of the dynamics of the liquid jets [10,11,13,17,18,20,5,6,8] due to the presence of centrifugal forces and based on the slender jet theory, where the aspect ratio for the fiber jet is assumed to be sufficiently small.

Decent et al. [5] did modeling and computation for the dynamics of an inviscid liquid jet, which was emerged from a rotating drum. The authors included effects of both gravity and surface tension and investigated both linear temporal and linear spatial instability of an asymptotically obtained steady basic state solution for the jet and used a multiple length scale in their stability analysis. For the linear temporal stability analysis, the authors found that axisymmetric mode can be unstable. Their linear spatial instability analysis also indicated instability of the jet.

Wallwork et al. [20] investigated a slender inviscid liquid jet, which was due to a rotating orifice in the liquid zone. The authors used a model for the slender curved inviscid jet in the presence of surface tension but assuming that the centerline of the jet was stationary. They determined the trajectory of the jet, and they also calculated the linear jet instability by applying a multiple-scale approach. They found, in particular, that such basic state becomes unstable with respect to infinitesimal travelling wave modes that grow along the jet.

Parau et al. [8] investigated the evolution of a disturbance on a rotating, slender jet, which emanated from an orifice. Using an asymptotic approach, they determined the equations for their modeling liquid jet. They determined, in particular, the steady solution for the liquid jet and found the effect of viscosity on the trajectory of the centerline and the radius of the jet. They also carried out numerical simulation to examine the break-up of the liquid jet. Decent et al. [6] studied the trajectory and linear stability of a spiraling liquid viscous jet using a Newtonian fluid. However, their developed model did not produce any effect of viscosity on their leading order steady state solution to the viscous jet flow. Their linear stability analysis was based

E-mail address: daniel.riahi@utrgv.edu.

http://dx.doi.org/10.1016/j.ijnonlinmec.2017.03.004
Received 19 February 2017; Accepted 5 March 2017
Available online 09 March 2017
0020-7462/

on the inclusion of disturbances of infinitesimal amplitude superimposed on an asymptotically determined steady state solution. They found, in particular, that by including the higher order terms in their governing system, then increasing Newtonian viscosity has the effect of making their jet solution more tightly coiled.

Padron et al. [10] did modeling and computation of a two-dimensional FS for the inviscid rotating jet and in the presence of surface tension. They developed a two-dimensional inviscid modeling system from the governing Euler and Continuity equations [21] and the associated boundary conditions for FS system. They solved numerically their modeling system and determined, in particular, fiber trajectory and the fiber radius versus arc length and for different values of the Weber number, which takes into account the surface tension effect, and the Rossby number, which takes into account the rotational effect. Their results indicated, in particular, that jet radius can decrease with increasing the rotation rate, while trajectories contract with increasing surface tension.

More recently Taghavi and Larson [17] investigated steady state problem for Newtonian jet flow formation by centrifugal spinning. They developed a so-called regularized thin-fiber modeling approach that made the steady state solution of the jet be affected by the presence of viscous effect. Taghavi and Larson [18] corrected a number of errors that existed in their earlier paper [17] and included a revised version of several figures for the jet speed and trajectory.

All the theoretical and computational studies so far on the subject of rotating jets which were described in the previous paragraphs were done for the Newtonian inviscid or viscous fluid flow cases. In the present study we consider the polymeric fiber jet during FS process by making use of a phenomenological viscosity model for the polymeric non-Newtonian fluid. As compared to the Newtonian fluid flow case, two features of the non-Newtonian polymeric fluid are notable [4]. The first one is due to the variation of the elongational viscosity of the polymer fluid, which can create the so-called extension thinning or extension thickening that can be seen as purely viscous and is possible to be represented by a generalized Newtonian viscosity as a function of the strain rate. The second feature is called strain hardening that reflects the polymer fluid memory and is a viscoelastic behavior. In the present paper we simplify the rheological behavior of the polymeric fluid by considering only the viscous aspects of the extensional rheology of the polymeric fluid. We, thus, follow an approach that is based on the type of empirical formula proposed first by Song and Xia [16] and later applied by Feng [7] for an electrified non-Newtonian jet. We develop the polymeric fluid model for the nonlinear rotating, slender jet by first deriving the governing equations and the boundary conditions for such fiber jet flow, and then determine the steady nonlinear solutions and their stability versus infinitesimal travelling wave perturbations. We find some interesting results. In particular, we find as compared to the Newtonian viscous fluid case, presence of the extensional thinning of the polymeric fluid, which promotes stretching, can make the fiber jet radius and speed smaller and higher, respectively, and the jet's stretching rate can be increased with the jet relaxation time. In addition, growing travelling wave perturbations in time only or in space only can decay to zero for the present polymeric fiber jet.

2. Formulation and modeling

The present investigation of the nonlinear rotating polymeric fiber jet flow is based on the original governing equations for the momentum and mass conservation [4] in a rotating frame of reference that is attached and embedded on the rotating spinneret (Fig. 1) of the forcespinning (FS) system. During the FS process, the produced fibers are curved due to the rotational forces (centrifugal and Coriolis forces) and depending on the values of the parameters will either break up due to instability or be collected at the end by a collector (Fig. 1) [12].

The nonlinear governing equations of continuity and momentum equations in the rotating frame are

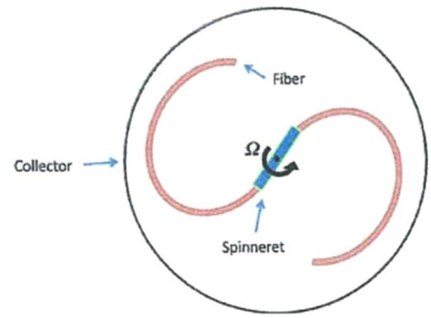

Fig. 1. Forcespinning schematic. (The author acknowledges with thanks ASME as original publisher of this figure as well as Fig. 2 that granted him permission to include and print from "On 2D Forcespinning Modeling", by Simon Padron, Dumitru I. Caruntu & Karen Lozano. Paper Number IMECE2011-64823).

$$\nabla .\, \mathbf{u} = 0, \tag{1a}$$

$$\partial \mathbf{u}/\partial t + \mathbf{u}.\, \nabla \mathbf{u} = (-1/\rho)\nabla P + (1/\rho)\nabla .\, \mathbf{T} - \omega \times (\omega \times \mathbf{r}) - 2\omega \times \mathbf{u}, \tag{1b}$$

where $\mathbf{u}$ is the relative velocity vector of the fiber jet, P is the pressure, $\mathbf{T}$ is the stress tensor, ρ is the density of the melt or solution, t is the time variable, $\boldsymbol{\omega}$ is the angular velocity vector with magnitude Ω of the rotating spinneret whose orifice emits fiber jet from the melt or solution in the fluid container [10,12], and $\mathbf{r}$ is a position vector of a point on the fiber. We consider a Cartesian coordinate system (X, Y, Z) which is attached to the rotating spinneret and is referred to here as the rotating coordinated system. This coordinate system is fixed relative to the rotating spinneret. Similar to the case studied in Wallwork et al. [20], we consider a two-dimensional model under the condition that the gravity force is negligible as compare to the rotating forces for the FS system, and the fiber jet's arc length condition that needs to be satisfied is given below

$$(\partial \mathbf{X}/\partial s)^2 + (\partial \mathbf{Z}/\partial s)^2 = 1, \tag{1c}$$

where s is the arc length along the jet's centerline. Thus, as in the work due to Wallwork et al. [20], it is assumed that the components of a position vector of a point on the jet's centerline with respect to the rotating Cartesian coordinate (X, Y, Z) system are $\mathbf{X}$, $\mathbf{0}$ and $\mathbf{Z}$, respectively.

The Eqs. (1a)–(1c) need to be considered subject to the fiber jet's boundary conditions. The jet's free surface kinematic and dynamic boundary conditions (Chahhabra and Richardson, 2008) are given below

$$\partial \beta/\partial t + \mathbf{u}.\, \nabla \beta = 0,\ \beta \equiv n - R(s, \phi, t), \tag{1d}$$

$$(\mathbf{T} - P\mathbf{I}).\, \mathbf{n} = -\sigma \kappa \mathbf{n}, \tag{1e}$$

where n and ϕ are the radial variable and the azimuthal angle, respectively, representing the variables of the polar coordinate in a plane perpendicular to the centerline of the jet, $\mathbf{n}$ is a unit normal vector perpendicular to the jet's surface boundary pointing out of the jet, $\mathbf{I}$ is a unitary matrix, R is the radius of the jet, σ is surface tension, and $\kappa \equiv \nabla .\mathbf{n}$ is twice mean curvature of the jet boundary. Similar to the treatment in Wallwork et al. [20], we presented these boundary conditions (1d) and (1e) in terms of independent variables of a local orthogonal curvilinear coordinates (s, n, ϕ). In addition, the following main boundary conditions at the orifice where jet exits [10,20] need to be satisfied

$$\mathbf{X} = \mathbf{Y} = \mathbf{Z} = \partial \mathbf{Y}/\partial s = \partial \mathbf{Z}/\partial s = 0,\ \ \partial \mathbf{X}/\partial s = 1,\ u = U,\ R = r_o \text{ at } s = 0, \tag{1f}$$

where U is the centerline velocity of the jet at the exit section and r_o is the radius of the orifice at the jet exit section.

As in the experimental observation and configuration for FS [10,12], we consider the rotating jet with curved centerline, and as in