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A method to solve the efficiency-accuracy trade-off of multi-harmonic balance calculation of structures with friction contacts

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ABSTRACT

The steady-state nonlinear forced response of systems with frictional damping can be computed in the frequency domain through the Harmonic Balance Method (HBM). A critical point is the selection of the number of harmonic terms used to represent the solution. In linear systems, this number is easily determined by the harmonic content of the forcing function (e.g. mono-harmonic). However, if nonlinearities are present, higher order harmonics may need to be included to ensure a proper representation of friction forces and displacements, with a detrimental effect on the computational time.

The paper presents a novel method to solve the efficiency-accuracy trade-off of harmonic selection for nonlinear systems. This method warns the user whenever the number of retained harmonic terms is inadequate. As a result, it enables the user to run the simulation with a low number of retained harmonics, e.g. designers are typically interested in the first harmonic of the solution. The calculation is repeated with a larger harmonic support only when strictly necessary to keep the error below a user-defined threshold.

The method is first compared to existing adaptive HBM techniques, highlighting its novel contributions. It is then carefully validated against high-order multi-harmonic calculations and against direct time integration. Its performance in terms of accuracy vs. computational time is highlighted. The method is then implemented in a state-of-the-art numerical tool for the design of underplatform dampers for turbine blades. Finally, its outcome is compared with experimental results.

1. Introduction

Forced vibrations of blades in power turbines still represent one of the most common causes of failure. The different parts of a turbine are connected together by interfaces. One of the challenges faced by turbine designers is to correctly design these interfaces to provide adequate friction damping to the system. The joints can be optimized in order to exploit the energy dissipated at the contact to limit the structure vibrations. The joints can be a part of the bladed disk (blade root joints [1,2], snubber [3] and shrouds [4]) or can be purposely added to the system (underplatform dampers [5], ring dampers [6]).

One of the most important parts of the design of systems characterized by frictional damping is the numerical prediction of the nonlinear forced response. Unfortunately this task cannot be performed by commercial finite element (FE) solvers in a reasonable amount of time. Rather it requires the development of custom codes capable of computing the amount of frictional damping for a given excitation condition. Several authors [7–10] have given their contribution in this field by taking into account the actual stick-slip, lift off states of the contact in order to determine the effect of friction on the resonant frequency (stiffness contribution) and on the resonant amplitude (damping contribution). Most of these papers deal with modeling underplatform dampers (UPDs), metal devices compressed by the centrifugal force under the blade platforms. The relative motion between adjacent blades causes the damper to slip, and therefore to dissipate energy. The common idea is to model both the blade and the UPD using FE and to introduce contact elements [4] between them.

This paper presents the results of a numerical tool developed to be used at design stage when the shape of the UPD is still not known. The tool is required to be very flexible using the minimum number of equations in order to be run several times at design stage. The method adopted for the solution of the equations of motion is the harmonic balance method (HBM) [11]. The number of equations to be solved is given by the number of contact (potentially nonlinear) degrees of freedom and the number of harmonic terms retained in the calculation.

A trade-off between accuracy of the solution and speed of computation exists: the number of retained harmonic terms should guarantee a proper representation of friction forces and, at the same time, be kept to a minimum to limit the number of algebraic variables. This trade-off is typically realized through cumbersome convergence studies [4,9,12],

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Nomenclature

Abbreviations

AHBM adaptive harmonic balance method
AFT alternating frequency-time method
DOF(s) degree(s) of freedom
DTI direct time integration
HBM harmonic balance method
JAA Jacobian alert algorithm
UPD underplatform damper

Variables

Ndof degrees of freedom retained in HBM computation
EC evaluation criterion
En strain energy
Err error indicator/evaluation criterion
t time
m generic frequency step
n_H number of harmonics used in the HBM (selected by the user)
$\tilde{n}_H$ number of harmonics used in the HBM (that ensure a highly accurate representation of friction forces and displacements even in severely nonlinear cases)
$\Delta J_{\%}$ relative norm indicator, to evaluate similarity between two matrices

Vectors and matrices

$\mathbf{M}$ mass matrix
$\mathbf{C}$ damping matrix
$\mathbf{K}$ stiffness matrix
$\mathbf{D}$ dynamic stiffness matrix
$\mathbf{q}$ generic displacement vector (either in time or in frequency domain retaining n_H harmonics)
$\tilde{\mathbf{q}}$ displacement vector in the frequency domain obtained retaining $\tilde{n}_H$ harmonics
$\mathbf{F}$ generic force vector
$\mathbf{r}$ vector of residuals
$\hat{\mathbf{K}}$ matrix of partial derivatives of contact forces with respect to displacements
$\mathbf{J}$, $\tilde{\mathbf{J}}$ Jacobian matrix, obtained using and respectively $\mathbf{q}$ and $\tilde{\mathbf{q}}$
$\boldsymbol{\Delta}\tilde{\mathbf{q}}$ correction applied to vector $\mathbf{q}$ by existing harmonic selection techniques to take into account higher harmonics contribution
$(\mathbf{q} + \boldsymbol{\Delta}\mathbf{q})$ approximation of vector $\tilde{\mathbf{q}}$ obtained using $\mathbf{J}$, implemented by novel technique JAA
$\mathbf{T}$ vector of tangential force at the contact
$\mathbf{t}$ vector of tangential displacement at the contact (perpendicular to the damper axis)
$\mathbf{w}$ vector of tangential displacement at the contact (parallel to the damper axis)
$\mathbf{n}$ vector of normal displacement at the contact
$\mathbf{id}$ vector identifying a given set of physical degrees of freedom
$\boldsymbol{\Gamma}$ matrix, close to the identity matrix, measure of the similarity between two matrices

Subscripts

D damper
B blades
C contact
CD contact, applied at the damper center of mass
CB contact, applied at the blades contact nodes
CL centrifugal load
EXT external excitation
HE harmonic excitation
$|_i$ iteration i
L1 L2 R contact points ID, as shown in Fig. 1b
L N linear, nonlinear
OUT output
p predicted

Superscripts

h, k harmonic indexes
n_H maximum harmonic index retained in nonlinear calculation
$\tilde{n}_H/n_H$ harmonics in the $[0\text{–}\tilde{n}_H]$ interval not retained in nonlinear HBM calculation, i.e. $[0\text{-}n_H]$

where the same forced response is computed many times by progressively increasing the number of retained harmonics. More recently, several harmonic selection algorithms [13–17] have been proposed. The main goal of this paper is to propose and validate a novel method to solve this trade-off and compare it to existing methods.

This novel method warns the user whenever the number of retained harmonic terms is inadequate – i.e. whenever the solution would significantly change if the number of harmonic terms were to increase. As a result, it enables the user to run the simulation with a low number of retained harmonics and to add harmonics only when strictly necessary. It should be noted that the tool here proposed evaluates the adequacy of the harmonic terms retained in the non-linear calculation without the need for repeated iterative calculations.

This novel method is only one of the contributions of this paper aimed at maximizing the computation speed of the design tool without sacrificing its accuracy. The complete list of the main features of the design tool is reported below.

1) Each damper is modelled as a rigid body due to its bulky structure. It has inertia properties and six degrees of freedom at the center of mass. This choice has two main benefits:
   – the contact points can be conveniently chosen on the blade platforms, according to an estimated damper geometry even if at the design stage a finite element model of the damper is not yet available;
   – the displacements and the forces at the contact points are transferred to the damper center of mass. In this way the degrees of freedom associated to the damper are always limited to six whatever the number of chosen contact points.
2) A Craig Bampton reduction [18] of blades mass and stiffness matrices is performed. In it only the contact DOFs (plus output and input nodes of interest) are retained, while all other liner DOFs are approximated with a set of linear normal modes computed by imposing proper boundary conditions.
3) The resulting blades DOFs (reduced linear plus contact/nonlinear) are further condensed [10], i.e. linear DOFs are expressed as a function of contact (potentially nonlinear) DOFs. As a result, non-contact DOFs are excluded from the equations; full accuracy is preserved and the size of the system to be solved through HBM is limited by the harmonic order and the number of contact DOFs only.
4) The Jacobian matrix of the system, required by the iterative scheme, is calculated analytically as described by [4], rather than numerically. This decreases enormously the computation time.