International Journal of Non-Linear Mechanics 92 (2017) 84–91

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Normal stress effects in the gravity driven flow of granular materials

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ARTICLE INFO

Keywords:
Granular materials
Normal-stress effects
Maximum packing
Shear-thinning fluid
Second grade fluids
Continuum mechanics

ABSTRACT

In this paper, we study the fully developed gravity-driven flow of granular materials between two inclined plates. We assume that the granular materials can be represented by a modified form of the second grade fluid where the viscosity depends on the shear rate and volume fraction and the normal stress coefficients depend on the volume fraction. We also propose a new isotropic (spherical) part of the stress tensor which can be related to the compactness of the (rigid) particles. This new term ensures that the rigid solid particles cannot be compacted beyond a point, namely when the volume fraction has reached the critical/maximum packing value. The numerical results indicate that the newly proposed stress tensor has obvious and physically meaningful effects on both the velocity and the volume fraction fields.

1. Introduction

Granular materials are discrete solid (macroscopic) particles of different shapes and sizes with interstices filled with a fluid. Granular materials occur in many natural processes, such as the flow of sand, snow and ice. In industrial applications, water and granular materials (grains, powders, coals, etc. [1]) are the first and the second mostly used materials. A granular medium does not behave as a classical solid continuum since it deforms or takes the shape of the vessel containing it; it is not exactly a liquid, even though it can flow, for it can be piled into heaps; and it is not a gas since it will not expand to fill the container. In many ways the bulk solids resemble non-Newtonian (non-linear) fluids [2]. The behavior of granular materials, in general, is determined by interparticle cohesion, friction, collisions, etc [3]. A granular medium includes granular powders and granular solids with components ranging in size from about 10 μm up to 3 mm. According to many researchers, a powder is composed of particles up to 100 μm (diameter) with further subdivision into ultrafine (0.1–1.0 μm), superfine (1–10 μm), or granular (10–100 μm) particles, whereas a granular solid consists of particles ranging from about 100 to 3000 μm [Brown and Richards [4]].

Granular materials present a multi-disciplinary field; they can be studied from different perspectives. For example, in order to characterize their rheological behavior, one can study the mechanics (or physics) of these complex materials by performing experiments, which are oftentimes very complicated. Recent review articles by Savage [5], Hutter and Rajagopal [6], and de Gennes [7], and books by Mehta [8], Duran [9], and Antony et al. [10] point to many of the important issues in modeling granular materials. From a theoretical perspective, there are two distinct, yet related methods that can be used: the statistical theories and the continuum theories. There are many recent review articles discussing the statistical theories [Herrmann [11], Herrmann and Luding [12]], and the kinetic theories as applied to granular materials [Goldhirsch [13] and Boyle and Massoudi [14]].

In this paper, we model the granular materials as a single phase continuum, ignoring the effects of the interstitial fluid. However, it should be pointed out that in many applications, the effect of the interstitial fluid is important and the problem should be studied using a multi-component method [see Rajagopal and Tao [15], Massoudi [16]]. In Section 2, we will present the governing equations. In Section 3, we will discuss the constitutive equations. In Section 4, we introduce the geometry and kinematics of the problem. In Section 5, we present numerical results through a parametric study of the non-linear ordinary differential equations for the gravity driven flow of the granular materials between two inclined plates.

2. Governing equations

We model the granular materials as a single component non-linear fluid. That is, we ignore the presence of the interstitial fluid and assume

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http://dx.doi.org/10.1016/j.ijnonlinmec.2017.03.016
Received 26 January 2017; Accepted 13 March 2017
Available online 21 March 2017
0020-7462/ Published by Elsevier Ltd.

that the assembly of the densely-packed particles form a continuum. Let $\boldsymbol{X}$ denote the position of this continous body. The motion can be represented as

$$\boldsymbol{x} = \chi(\boldsymbol{X}, t) \tag{1}$$

while the kinematical quantities associated with the motion are

$$\boldsymbol{v} = \frac{d\boldsymbol{x}}{dt} \tag{2}$$

$$\boldsymbol{D} = \frac{1}{2}\left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}} + \left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}\right)^T\right) \tag{3}$$

where $\boldsymbol{v}$ is the velocity field, $\boldsymbol{D}$ is the symmetric part of velocity gradient, and $\frac{d}{dt}$ denotes differentiation with respect to time holding $\boldsymbol{X}$ fixed and superscript 'T' designates the transpose of a tensor. The bulk density field, ρ, is

$$\rho = \rho_0\phi \tag{4}$$

where ρ_0 is the pure density of granular materials, in the reference configuration; ϕ is the volume fraction, where $0 \le \phi < \phi_{max} < 1$. The function ϕ is represented as a continuous function of position and time; in reality, ϕ is either one or zero at any position and at any given time. That is, in a sense, we have performed a homogenization process (see Collins [17]) whereby the shape and size of the particles in this idealized body have disappeared except through the presence of the volume fraction. For details see Massoudi [18] and Massoudi and Mehrabadi [19]. In reality, ϕ is never equal to one; its maximum value, generally designated as the maximum packing fraction, depends on the shape, size, method of packing, etc.

Having defined the basic kinematical parameters, we now look at the conservation equations. In the absence of any thermo-chemical and electro-magnetic effects, the governing equations for the flow of a single-component material are the conservation equations for mass, linear momentum, and angular momentum [20]. As we are only considering a purely mechanical system, the energy equation and the entropy inequality are not considered.

2.1. Conservation of mass

The conservation of mass is:

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0 \tag{5}$$

where $\partial/\partial t$ is the derivative with respect to time, div is the divergence operator.

2.2. Conservation of linear momentum

Let $\boldsymbol{T}$ represent the Cauchy stress tensor for the granular materials. Then the balance of the linear momentum is:

$$\rho\frac{d\boldsymbol{v}}{dt} = div\boldsymbol{T} + \rho\boldsymbol{b} \tag{6}$$

where $\frac{d\boldsymbol{v}}{dt} = \frac{\partial \boldsymbol{v}}{\partial t} + (grad\boldsymbol{v})\boldsymbol{v}$ and $\boldsymbol{b}$ stands for the body force.

2.3. Conservation of angular momentum

$$\boldsymbol{T} = \boldsymbol{T}^T \tag{7}$$

The above equation implies that in absence of couple stresses the Cauchy stress tensor is symmetric. The constitutive relation for the stress tensor need to be specified before any problem can be solved. In the next section, we will discuss this issue.

3. Constitutive equation: the stress tensor

Most granular materials exhibit two unusual and peculiar characteristics: (i) normal stress differences, and (ii) yield criterion.[1] Reynolds [21] observed that in a bed of closely packed particles, the bed must expand if a shearing motion is to occur; this occurs in order to increase the volume of the voids. Reynolds [22] called this phenomenon 'dilatancy' and he was able to describe the capillary action in wet sand. The concept of dilatancy, which in a larger context, is related to the normal stress differences in non-linear materials, is related the expansion of the void volumes in a packed arrangement when subjected to a deformation. This can be explained for an idealized case: in a bed of closely packed spheres, for a shearing motion to occur, for example, between two flat plates, the bed must expand by increasing its void volume. Reiner [23,24] used a non-Newtonian model to predict dilatancy in wet sand [see Massoudi [25,26]]. Perhaps the simplest constitutive equation which can describe the normal stress effects in non-linear fluids (related to phenomena such as 'die-swell' and 'rod-climbing', which are manifestations of the stresses that develop orthogonal to planes of shear) is the second grade fluid, or the Rivlin-Ericksen fluid of grade two [Rivlin and Ericksen [27], Truesdell and Noll [28]]. This model is a special case of fluids of differential type [Dunn and Rajagopal [29]]. For a second grade fluid, the Cauchy stress tensor is given by [30]:

$$\boldsymbol{T} = -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2 \tag{8}$$

where p is the indeterminate part of the stress due to the constraint of incompressibility, μ is the coefficient of viscosity which may depends on shear rate, volume fraction, pressure, temperature, etc. [31–33], and α_1 and α_2 are material moduli which are commonly referred to as the normal stress coefficients [34]. The kinematical tensors $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are defined through

$$\begin{cases} \boldsymbol{A}_1 = \boldsymbol{L} + \boldsymbol{L}^{\mathrm{T}} \\ \boldsymbol{A}_2 = \frac{\mathrm{d}\boldsymbol{A}_1}{\mathrm{dt}} + \boldsymbol{A}_1\boldsymbol{L} + \boldsymbol{L}^{\mathrm{T}}\boldsymbol{A}_1 \\ \boldsymbol{L} = grad\boldsymbol{v} \end{cases} \tag{9}$$

where $\boldsymbol{L}$ is the velocity gradient. The thermodynamics and stability of fluids of second grade have been studied in detail by Dunn and Fosdick [35]. They concluded that if the fluid is to be thermodynamically consistent in the sense that all motions of the fluid meet the Clausius-Duhem inequality and that the specific Helmholtz free energy be a minimum in equilibrium, then

$$\mu \ge 0 \tag{10)a}$$

$$\alpha_1 \ge 0 \tag{10)b}$$

$$\alpha_1 + \alpha_2 = 0 \tag{10)c}$$

It is known that for many non-linear fluids which are assumed to follow Eq. (8), the experimental values reported for α_1 and α_2 do not satisfy the restriction[2] of Eqs. (10)b and (10)c. For further details on this and other relevant issues in fluids of differential type, we refer the reader to the review article by Dunn and Rajagopal [29]. For some applications, such as flow of ice or coal slurries, where the fluid is known to be shear-thinning (or shear-thickening), modified (or generalized) forms of the second grade fluid have been proposed [see Man [36], Massoudi and Vaidya [37], Man and Massoudi [38]].

In this paper, we assume that the (flowing) granular materials can be modeled as a non-linear fluid (of second grade type) capable of exhibiting normal stress effects, where the shear viscosity depends on the volume fraction and the shear rate and the normal stress coefficients

[1] In this paper, we do not discuss the yield stress.

[2] In an important paper, Fosdick and Rajagopal [47] show that irrespective of whether $\alpha_1 + \alpha_2$ is positive, the fluid is unsuitable if α_1 is negative.

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