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Controlled synchronization at the existence limit for an excited unbalanced rotor

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ABSTRACT

The synchronization of a controlled unbalanced rotor with a viscoelastically mounted supporting body and force-excitation is studied. The existence and stability conditions for the synchronous regime of motion are derived for a general control law by the method of direct separation of motion. Then a control law is developed using speed gradient method in order to transfer maximum energy from the excitation to the rotor. The free parameters of the control law are derived in such a way that the controlled synchronization is stable at the existence limit.

1. Introduction

The effect of synchronization, also called lock-in effect or synergy, has been known over decades in fields like physics, biology, chemistry and psychology, based on a natural principle acting from quantum to celestial level [1,2]. In the mechanical sense two oscillators with slightly different periods can synchronize due to a weak coupling on the same periodic time. In 1948 this effect also was observed for coupled unbalanced rotors during experiments [3]. Since then many applications have been developed including synchronized unbalanced rotors driven by electric motors with base- or force-excitation as well as coupled multiple unbalanced rotors. Based on approaches with uncontrolled motors [4,5], called self-synchronization, research was done to improved and ensure the stability using control algorithms. In most of these cases the stable synchronous state itself was the aim of control [6,7]. An amazing byproduct of synchronization is the transfer of energy from the faster oscillator to the slower one that can be exploited for energy harvesting or vibration reduction in technical applications.

The entrainment of a self-sustained oscillator by an external force, also called capture effect, can be seen as the simplest case of synchronization [1]. In this sense the synchronization between an external force excitation and the motion of an unbalanced rotor is studied in this contribution. A control torque will be applied to the rotor. The aim is to define a control law for a stable energy transfer from the excitation force to the rotor in the synchronous state. To this end the control parameters will be derived fulfilling the existence and stability conditions for the synchronous state that are obtained by using the method of direct separation of motion [8].

The paper is organized as follows: Outgoing from the equations of motion the 0th-order approximations of the existence and stability conditions for the controlled synchronization with a general control law using the method of direct separation of motion are derived in Section 2. Section 3 contains the definition of a control law for the motor torque, depending on the total energy of the system, using speed gradient method. Then the free parameters for a simplified control law, only depending on the kinetic energy of the rotor, are derived fulfilling the existence and stability conditions at the existence limit. Results from numerical simulations are presented in Section 4.

2. Existence and stability conditions for a force-excited unbalanced rotor

The considered system consists of a rotor driven by an electric motor (moment of inertia J, control torque M) with an eccentric mass (eccentricity e, mass m_e) on a viscoelastically mounted supporting body (stiffness c, damping coefficient d, mass m) with a force excitation $F(t) = \hat{F}\sin\Omega t$ with the excitation frequency $\Omega = \mathrm{const}$ according to Fig. 1. The coordinates of the system are the displacement of the supporting body x and the rotor angle φ. Gravitational effects are not considered here.

2.1. Equations of motion

The total energy of the system is given by

$$H(\varphi, \dot{\varphi}, x, \dot{x}) = H_1(\dot{\varphi}) + H_2(\varphi, \dot{\varphi}, \dot{x}) + H_3(x, \dot{x}) \quad (1)$$

containing the kinetic rotor energy

$$H_1(\dot{\varphi}) = \frac{1}{2}J_e\dot{\varphi}^2, \quad J_e = J + m_e e^2 \quad (2)$$

the kinetic energy of interaction between the rotor and the supporting

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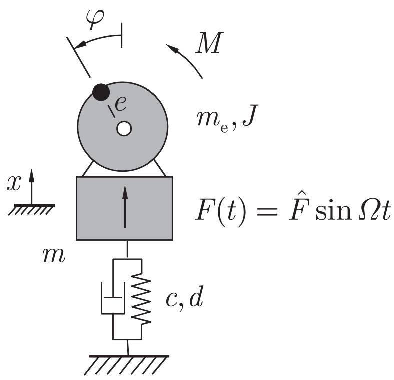

Fig. 1. Unbalanced rotor with control torque M and force-excitation.

body

$$H_2(\varphi,\dot{\varphi},\dot{x}) = -m_e\dot{x}\dot{\varphi}e\sin\varphi \tag{3}$$

and the kinetic and potential energy of the supporting body itself

$$H_3(x,\dot{x}) = \frac{1}{2}cx^2 + \frac{1}{2}(m+m_e)\dot{x}^2. \tag{4}$$

With the Lagrangian $L = H(\varphi,\dot{\varphi},x,\dot{x}) - cx^2$ the Lagrange equations of the second kind yield the equations of motion in the form

$$(m+m_e)\ddot{x} + d\dot{x} + cx = \hat{F}\sin\Omega t + m_e e(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi), \tag{5}$$

$$J_e\ddot{\varphi} = m_e e\ddot{x}\sin\varphi + M(s,t) \tag{6}$$

with the total inertia of the rotor $J_e = J + m_e e^2$ and the time derivative defined as $\dot{\cdot} \equiv \frac{\mathrm{d}}{\mathrm{d}t}$. For (6) it is assumed that a control law in the form $M = M(s,t)$, with the state vector $s = [\varphi,\dot{\varphi},x,\dot{x}]^{\mathrm{T}}$, can be found. For the method of direct separation of motion two time scales are introduced, the slow time t and the fast dimensionless time $\tau = \Omega t$, $\Omega \gg 1$. For this purpose the equations of motion are reparametrized with the fast time $\tau = \Omega t$, defining $' \equiv \frac{\mathrm{d}}{\mathrm{d}\tau} = \frac{1}{\Omega}\frac{\mathrm{d}}{\mathrm{d}t}$, thus

$$(m+m_e)\Omega^2 x'' + d\Omega x' + cx = \hat{F}\sin\tau + m_e e(\Omega^2\varphi''\sin\varphi + \Omega^2\varphi'^2\cos\varphi), \tag{7}$$

$$J_e\Omega^2\varphi'' = m_e e\Omega^2 x''\sin\varphi + M(s,t,\tau). \tag{8}$$

The normal forms of the differential equations read

$$x'' + 2\bar{\delta}x' + \bar{\omega}_x^2 x = \bar{\bar{F}}\sin\tau + \frac{\varepsilon}{m+m_e}(\varphi''\sin\varphi + \varphi'^2\cos\varphi), \tag{9}$$

$$\varphi'' = \frac{\varepsilon}{J_e}x''\sin\varphi + \overline{M}(s,t,\tau) \tag{10}$$

with

$$\bar{\delta} = \frac{d}{2(m+m_e)\Omega} \equiv \frac{\delta}{\Omega},\quad \bar{\omega}_x^2 = \frac{c}{(m+m_e)\Omega^2} \equiv \frac{\omega_x^2}{\Omega^2},\ \varepsilon = m_e e,$$
$$\bar{\bar{F}} = \frac{\hat{F}}{2(m+m_e)\Omega^2},\quad \overline{M} = \frac{M}{J_e\Omega^2}. \tag{11}$$

In case of synchronization the mean value of the angular velocity of the rotor $\dot{\varphi}$ equals a rational multiple of the angular excitation frequency, $\frac{k}{p}\Omega$, called regime of type k/p with the natural numbers k, p. The separation ansatz for the rotation of the rotor

$$\varphi(t,\tau) = \frac{k}{p}\tau + \alpha(t) + \psi(t,\tau) \tag{12}$$

contains the constant average angular velocity $\frac{k}{p}\Omega$, the slowly changing phase angle $\alpha(t)$ and a rapidly changing periodic term $\psi(t,\tau)$ whose average value over one period in the fast time scale is zero. The average value of a function $f(t,\tau)$ over the period $2\pi p$ in the fast time scale is defined as

$$\frac{1}{2\pi p}\int_0^{2\pi p} f(t,\tau)\,\mathrm{d}\tau \equiv \langle f(t,\tau)\rangle, \tag{13}$$

where all components in the slow time scale t are assumed to be constant over one period in the fast time scale.

2.2. Approximation of the motion of the supporting body

Introducing the separation (12) into the equation of motion (9) yields the differential equation for stationary solutions of the phase angle $\alpha^0 = \text{const}$,

$$x'' + 2\bar{\delta}x' + \bar{\omega}_x^2 x = \bar{\bar{F}}\sin\tau + \varepsilon\overline{V}_{x,k/p}, \tag{14}$$

with

$$\varepsilon\overline{V}_{x,k/p} = \frac{\varepsilon}{m+m_e}\left(\psi''\sin\left(\frac{k}{p}\tau + \alpha^0 + \psi\right) + \left(\frac{k}{p} + \psi'\right)^2\cos\left(\frac{k}{p}\tau + \alpha^0 + \psi\right)\right). \tag{15}$$

For the fast motion of the supporting body $x(\tau)$ periodic solutions are searched. Assuming the parameter ε to be small, $\varepsilon \ll 1$, an asymptotic expansion of (14) is appropriate [9]. For this the ansatz for the fast motions $x(\tau)$, $\psi(t,\tau)$,

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \ldots,\qquad \psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \ldots, \tag{16}$$

will be introduced into (14). After the series expansion of (14) with respect to ε at $\varepsilon = 0$ the comparison of the coefficients of the powers of ε yields a system of differential equations for the displacement $x(\tau)$ of the supporting body

$$\varepsilon^0:\quad x''_0 + 2\bar{\delta}x'_0 + \bar{\omega}_x^2 x_0 = \bar{\bar{F}}\sin\tau\quad \varepsilon^1:$$
$$x''_1 + 2\bar{\delta}x'_1 + \bar{\omega}_x^2 x_1 = \overline{V}_{x,k/p,0}\quad \varepsilon^2:\quad x''_2 + 2\bar{\delta}x'_2 + \bar{\omega}_x^2 x_2 = \overline{V}_{x,k/p,1}\quad \vdots \tag{17}$$

with

$$\overline{V}_{x,k/p,0} = \overline{V}_{x,k/p}\big|_{\varepsilon=0} = \frac{1}{m+m_e}\left(\psi''_0\sin\left(\frac{k}{p}\tau + \alpha^0 + \psi_0\right) + \left(\frac{k}{p} + \psi'_0\right)^2\right.$$
$$\left.\cos\left(\frac{k}{p}\tau + \alpha^0 + \psi_0\right)\right)\overline{V}_{x,k/p,1} = \left.\frac{\partial\overline{V}_{x,k/p}}{\partial\varepsilon}\right|_{\varepsilon=0} = \cdots\quad\vdots \tag{18}$$

The first differential equation of (17) does not depend on the rotation angle of the rotor φ, thus its particular solution and by this the 0th-order approximation of the particular solution of (14) is directly obtained as

$$x_0(\tau) = -\bar{\bar{F}}(A_\delta\cos\tau - A_\omega\sin\tau) \tag{19}$$

by using the influence numbers

$$A_\omega = \frac{\bar{\omega}_x^2 - 1}{(\bar{\omega}_x^2 - 1)^2 + 4\bar{\delta}^2}, \tag{20}$$

$$A_\delta = \frac{2\bar{\delta}}{(\bar{\omega}_x^2 - 1)^2 + 4\bar{\delta}^2}. \tag{21}$$

2.3. Approximation of the rotor motion

For a solution of the perturbed equation of motion (10), the solution of the unperturbed differential equation in ψ with $\varepsilon = 0$ must be known. For this purpose the nonlinear control torque has to be linearized at the stationary angular velocity $\varphi' = \frac{k}{p}$ and the angle $\varphi = \frac{k}{p}\tau + \alpha$, thus