International Journal of Non–Linear Mechanics 91 (2017) 140–150

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Non-smooth model and numerical analysis of a friction driven structure for piezoelectric motors

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ARTICLE INFO

Keywords:
Non-smooth mechanical system
Frictional contact
Piezoelectric motors

ABSTRACT

In this contribution, typical friction driven structures are summarized and presented considering the mechanical structures and operation principles of different types of piezoelectric motors. A two degree-of-freedom dynamic model with one unilateral frictional contact is built for one of the friction driven structures. Different contact regimes and the transitions between them are identified and analyzed. Numerical simulations are conducted to find out different operation modes of the system concerning the sequence of contact regimes in one steady state period. The influences of parameters on the operation modes and corresponding steady state characteristics are also explored. Some advice are then given in terms of the design of friction driven structures and piezoelectric motors.

1. Introduction

Piezoelectric motors have been extensively studied and successfully applied to such areas as medical instruments [1], and consumer electronics [2] during the past few decades, with their outstanding features [3] of high torque at low speed, quick response, quiet operation and compact size. They usually achieve mechanical output through the frictional contact between the stator and rotor/slider under the help of the induced elastic vibration in the composite stator. According to their operation principles, piezoelectric motors are divided into two types. In the first type of piezoelectric motors, resonant vibration is induced in the stator. Traveling wave piezoelectric motors [4] and standing wave piezoelectric motors [5] fall into this type. On the other hand, quasi-static vibration is usually induced in the stator of the second type of piezoelectric motors. As a result the rotor/slider usually conducts a quasi-static motion sequence somewhat like a stepping motor. Stick slip motors [6] and inchworm motors [7] belong to this type.

According to the underlying physics, mathematical modeling of piezoelectric motors mainly focuses on two parts: mathematical description of the electro-mechanical coupled stator and describing model for the contact interface between the stator and rotor/slider. The multi-physics nature of the coupled stator brings about complexity in terms of the coupling between elasticity and electricity. Some fruitful attempts have been made to manage the complexity using direct analytical method [8], equivalent circuit method [9], and finite element analysis method [10]. The frictional contact between the stator and rotor/slider introduces strong nonlinearity and discontinuity into the model, especially when both the normal and the tangential contact are taken into consideration. Hitherto there have been numerous researches on the description of friction forces and their applications into piezoelectric motors. Hunstig et al. [11,12] proposed a systematic analytical model for stick slip motors by adopting the Coulomb friction model, and investigated the influences of excitation signals on motor performances. Zharii [13] developed a mathematical model of a wedge-type ultrasonic motor considering the regime of slip and gave detailed discussions. Lu et al. [14] extended the model, divided each cycle of the stator vibration into several stages, and established the equations of rotor motion for each stage. Though the models provide insights into the operation of piezoelectric motors, little has been addressed in terms of the influences of contact properties on motor operation.

In this contribution, two typical friction driven structures are put forward considering vibration of the stator and motion of the rotor in different piezoelectric motors. The second type of friction driven structure is modeled as a two degree-of-freedom mechanical system with one unilateral frictional contact. Dynamic equations of the system are developed in terms of the four identified frictional contact regimes. Simulations are conducted to recognize different operation modes of the friction driven structure regarding the number and order of different contact regimes in one steady state operation period. Furthermore, the influences of system parameters such as drive frequency, coefficient of friction and initial tilt angle of the converted

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http://dx.doi.org/10.1016/j.ijnonlinmec.2017.02.009
Received 5 January 2016; Received in revised form 16 February 2017; Accepted 16 February 2017
Available online 21 February 2017

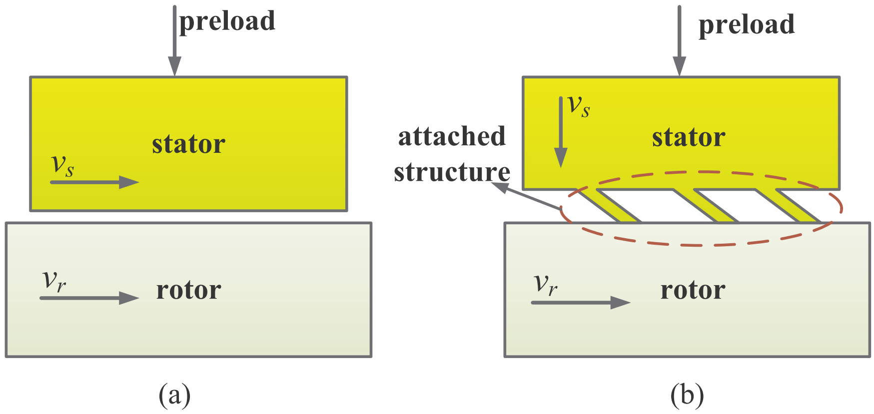

Fig. 1. Friction driven structures: the first type (a), and the second type (b).

rod on the steady state characteristics of the system are also evaluated.

2. Friction driven structure and its dynamic model

As described above, friction forces between the stator and rotor/slider play a key role in the operation of piezoelectric motors and largely determine the output performances, with the help of corresponding friction driven structures. According to different mechanical structures and operation principles of piezoelectric motors, two different types of friction driven structure can be extracted. For the first type, the direction of stator vibration is parallel to the resultant motion direction of rotor/slider, as shown in Fig. 1(a). For simplicity here in the presented figures, a slider is used to denote the movable part in a piezoelectric motor (usually rotor or slider though), but the slider is annotated with “rotor” following the convention of rotary motors. In fact, a rotor or a slider can be placed and annotated here interchangeably without affecting the modeling process. We then use the word “rotor” across this contribution for simplicity and consistency. Traveling wave piezoelectric motors and stick slip motors belong to this type. Notice that in traveling wave motors, though the vibration direction of the mass particles in the stator is perpendicular to the rotor motion direction, the traveling wave excited in the stator propagates in a direction parallel to the rotor motion direction. For the second type including most standing wave motors and inchworm motors, direction of the stator vibration is usually different from that of the rotor motion, as shown in Fig. 1(b). The generation of friction forces is achieved by the attached structure to the stator structure which couple the vibration of the stator and results in a tangential motion component.

The first type of friction driven structure is somehow similar to the belt conveyor system, and extensively studied [15–19]. It is relatively simple as the normal forces in the frictional contact is constant. However, in the second type of friction driven structure, the normal contact forces are time varying, which brings about great complexity in the analysis and simulation. Hence in this paper we focus on the modeling and analysis of the second type of friction driven structure. As shown in Fig. 2(a), the attached structure is usually in the form of cantilever beam and therefore can be replaced by a rigid link hinged to the stator with an extra supporting spring representing equivalent stiffness of the cantilever. As a result, the friction driven structure is converted into the system shown in Fig. 2(b), consisting of the stator, the rigid rod, the rotor block and the supporting spring.

Choosing the vertical displacement x_1 of the stator block, the rotation angle φ of the rod and the horizontal displacement x_2 of the rotor as the generalized coordinates of the system, the resultant kinetic and potential energy of the system are expressed as

$$\begin{cases} T = \frac{1}{2}(m + m_1)\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{6}ml^2\dot{\varphi}^2 + \frac{1}{2}ml\dot{\varphi}\dot{x}_1\cos\varphi \\ V = \frac{1}{2}kl^2(\sin\varphi_0 - \sin\varphi)^2 \end{cases}, \tag{1}$$

where m_1 is the mass of the stator, m is the mass of the rod, m_2 is the mass of the rotor, l is the length of the rod, k is the equivalent stiffness of the supporting spring, and φ_0 is the initial tilt angle of the rod. The normal and tangential contact forces between the rod and the rotor are denoted by λ_N and λ_T respectively and the external forces applied to the rotor m_2 and stator m_1 are represented by F_L and F_A respectively. The corresponding external work is

$$W_e = F_A(x_1 - x_{1i}) + F_L(x_2 - x_{2i}) + \lambda_N(l\sin\varphi_0 - l\sin\varphi - x_1) + \lambda_T(x_2 - l\cos\varphi). \tag{2}$$

where x_{1i} and x_{2i} represent the initial state of x_1 and x_2, respectively.

With the Lagrangian being $L = T - V$, dynamic equations of the simplified system turn out to be

$$\begin{cases} (m + m_1)\ddot{x}_1 + \frac{1}{2}ml\ddot{\varphi}\cos\varphi = F_A + \frac{1}{2}ml\dot{\varphi}^2\sin\varphi \\ m_2\ddot{x}_2 = \lambda_T - F_L \\ \frac{1}{2}m\ddot{x}_1\cos\varphi + \frac{1}{3}ml\ddot{\varphi} = kl\cos\varphi(\sin\varphi_0 - \sin\varphi) + \lambda_T\sin\varphi - \lambda_N\cos\varphi \end{cases}. \tag{3}$$

According to the operation principle of piezoelectric motors, vertical displacement x_1 of the stator is proportional to the applied voltage $U(t)$, which is usually in the form

$$U(t) = \frac{U_{pp}}{2}(1 - \cos wt), \tag{4}$$

where U_{PP} is amplitude of the applied voltage, w is angular frequency of the applied voltage that can be calculated as $w = 2\pi f$ with f being frequency of the applied voltage. Thus vertical displacement x_1 of the stator kinematics are known and expressed by

$$\begin{cases} x_1 = x_{10} - x_{10}\cos wt \\ \dot{x}_1 = wx_{10}\sin wt \\ \ddot{x}_1 = w^2x_{10}\cos wt \end{cases}, \tag{5}$$

where x_{10} is the amplitude of the displacement. As a result, Eq. (3) becomes

$$\begin{cases} m_2\ddot{x}_2 = \lambda_T - F_L \\ \frac{1}{2}m\ddot{x}_1\cos\varphi + \frac{1}{3}ml\ddot{\varphi} = kl\cos\varphi(\sin\varphi_0 - \sin\varphi) + \lambda_T\sin\varphi - \lambda_N\cos\varphi \end{cases} \tag{6}$$

Defining the dimensionless generalized coordinates

$$\begin{cases} u_1 = \frac{x_1}{l} \\ u_2 = \frac{x_2}{l} \end{cases} \tag{7}$$

and the dimensionless parameters