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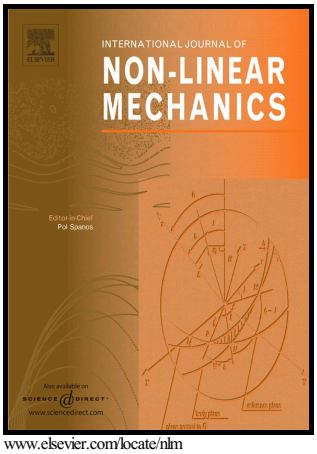


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# Non-local effects on the non-linear modes of vibration of carbon nanotubes under electrostatic actuation

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## Abstract

Carbon nanotubes (CNTs) based NEMS with electrostatic sensing/actuation may be employed as sensors, in situations where it is fundamental to understand their dynamic behaviour. Due to displacements that are large in comparison with the thickness and to the non-linearity of the electrostatic force, these CNT based NEMS operate in the non-linear regime. The knowledge of the modes of vibration of a CNT provides a picture of what one may expect from its dynamic behaviour not only in free, but also in forced vibrations. In this paper, the non-linear modes of vibration of CNTs actuated by electrostatic forces are investigated. For that purpose, a *p*-version finite element type formulation is implemented, leading to ordinary differential equations of motion in the time domain. The formulation takes into account non-local effects, which influence the inertia and the stiffness of CNTs, as well as the electrostatic actuation. The ordinary differential equations of motion are transformed into algebraic equations of motion via the harmonic balance method (HBM) and then solved by an arc-length continuation method. Several harmonics are considered in the HBM. The importance of non-local effects, combined with the geometrical non-linearity and with the action of the electrostatic force, is analysed. It is found that different combinations of these effects can result in alterations of the natural frequencies, variations in the degrees of softening or hardening, changes in the frequency content of the free vibrations, and alterations in the mode shapes of vibration. It is furthermore found that the small scale, here represented by the non-local theory, has an effect on interactions between the first and higher order modes which are induced by the geometrical and material non-linearities of the system.

Keywords: Carbon Nanotubes; electrostatic, non-local; non-linear modes of vibration.