International Journal of Non--Linear Mechanics 87 (2016) 38–42

Contents lists available at ScienceDirect

International Journal of Non–Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Neutral nano-inhomogeneities in hyperelastic materials with a hyperelastic interface model

Ming Dai[a,b], Peter Schiavone[b,*], Cun-Fa Gao[a]

[a] State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[b] Department of Mechanical Engineering, University of Alberta, Edmonton, Alberta T6G 1H9, Canada

ARTICLE INFO

Keywords:
Neutral inhomogeneity
Nanosized inhomogeneity
Hyperelastic interface effects
Nonlinear elasticity
Hyperelastic material

ABSTRACT

We examine the existence of neutral nano-inhomogeneities in a hyperelastic inhomogeneity-matrix system subjected to finite plane deformations when uniform (in-plane) external loading is imposed on the matrix. We incorporate nanoscale interface effects by representing the material interface as a separate hyperelastic membrane, perfectly bonded to the surrounding bulk material. We show that for any type of hyperelastic bulk material and practically any type of hyperelastic membrane representing the interface, neutral nano-inhomogeneities do exist but are necessarily *circular* in shape. We show further that the radius of the circular neutral nano-inhomogeneity is determined by the (uniform) external loading (which must be hydrostatic) and the respective strain energy density functions associated with the hyperelastic bulk and interface materials.

1. Introduction

Hyperelastic models have been used extensively in the analysis of a wide range of materials subjected to large deformation, for example, rubber [1,2], synthetic elastomers [3–5] and biological tissues [6–8]. When hyperelastic materials are considered at the nanoscale (for example, in the analysis of nano-rubbers [9]), high surface area to volume ratios require that continuum-based models of deformation further incorporate appreciable surface/interface effects resulting from surface/interface energies present but usually neglected at higher length scales. The development of analogous continuum-based models in the case of linearly elastic materials has been undertaken in a variety of different problems involving material inhomogeneities (see, for example, [10–13]) using the surface/interface theory attributed to Gurtin, Murdoch and co-workers [14,15]. Most recently, the Gurtin-Murdoch model has also been applied to the micromechanical analysis of hyperelastic materials containing nano-holes or nano–inhomogeneities in composites subjected to finite plane deformations (see, for example, [16,17]). In the Gurtin-Murdoch theory, the surface/interface contribution is modeled as a separate linearly elastic membrane subjected to small-strain deformation. The use of this model in the analysis of hyperelastic materials at the nanoscale is attractive in that it simplifies the resulting mathematics but is also considered by many as incompatible since the mechanics of hyperelastic bulk materials is associated with finite deformations. To address this deficiency, non-linear surface/interface models utilizing the idea of a hyperelastic membrane have been established in the literature (see, for example, [18–20]) and most recently employed in the stress analysis of a circular nano-cylinder under axial tension [21] and combined tension and torsion [22].

Despite the development of hyperelastic surface/interface models, their application in specific problems dealing with hyperelastic nanostructures have been few and far between and often restricted to very simple cases (see, for example, [21–23]). In this paper, we present a further application of a hyperelastic interface model (also referred to as a hyperelastic membrane-type imperfect interface) in the design of neutral nano-inhomogeneities in hyperelastic materials subjected to finite plane deformations. We remind the reader that the idea of 'neutrality' was introduced for linear elasticity by Mansfield [24] in designing a reinforced hole which eliminates any stress concentrations introduced by the hole and hence does not disturb the original stress field in the uncut body. The analogous idea of a neutral elastic inhomogeneity was introduced by Ru [25]. In contrast to the membrane-type imperfect interface model used here, a spring-layer type imperfect interface model (originally developed by Hashin [26]) has been used to construct elliptical neutral inhomogeneities in hyperelastic materials subjected to finite plane deformations [27]. However, it is worth noting that the results in [27] are restricted to a specific class of compressible hyperelastic materials of harmonic type [28]. In this paper, we seek to establish fundamental results in the design of neutral nano-inhomogeneities in general hyperelastic materials (whether compressible or incompressible) using a general hyperelastic (membrane-

* Corresponding author.
E-mail addresses: mdai1@ualberta.ca (M. Dai), P.Schiavone@ualberta.ca (P. Schiavone), cfgao@nuaa.edu.cn (C.-F. Gao).

http://dx.doi.org/10.1016/j.ijnonlinmec.2016.09.010
Received 4 May 2016; Received in revised form 23 September 2016; Accepted 23 September 2016
Available online 24 September 2016
0020-7462/© 2016 Elsevier Ltd. All rights reserved.

type) interface model to account for nanoscale effects.

Our paper is organized as follows. In Section 2, we introduce a hyperelastic inhomogeneity-matrix system with interface effects described by a general hyperelastic membrane. In Section 3, we establish necessary and sufficient conditions guaranteeing neutrality in terms of parameters describing the shape and size of the inhomogeneity. Our results are summarized in Section 4.

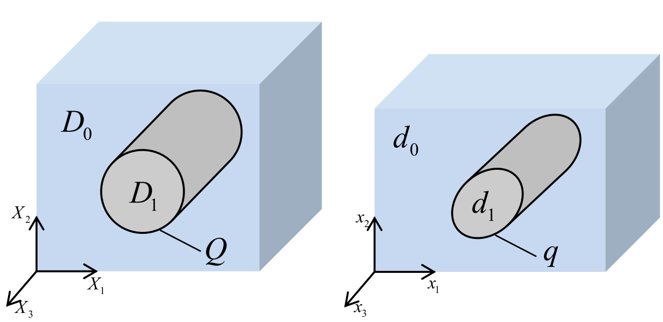
2. Problem formulation

In the theory of elasticity, a neutral inhomogeneity is defined as a foreign body which can be introduced into a host matrix without disturbing the existing stress field in the matrix. For a linearly isotropic inhomogeneity-matrix system with uniform interface effects described by the Gurtin-Murdoch model [14,15], it is known that circular inhomogeneities [29] and only circular inhomogeneities [30] admit neutrality when the system is subjected to plane deformations and the matrix to uniform external loading. Our purpose here is to examine the corresponding neutrality of inhomogeneities in the presence of interface effects in hyperelastic materials subjected to finite plane (strain) deformations.

We refer to Fig. 1 in which the axes of the (X_1, X_2, X_3)- and (x_1, x_2, x_3)- Cartesian coordinate systems are aligned as shown. We consider an isotropic hyperelastic inhomogeneity-matrix system in which the material interface is modeled as a separate isotropic hyperelastic membrane with vanishing thickness (and thus resists only stretching - no bending resistance), perfectly bonded to both the inhomogeneity and its surrounding matrix [18–23]. Further, the interface is represented by a cylinder parallel to the X_3-axis, having cross-section described by the curve Q in the X_1–X_2 plane. The inhomogeneity-matrix system undergoes plane strain deformation (neglecting rigid translation and rotation) in the X_1–X_2 plane. Consequently, the principal stretches in the matrix, inhomogeneity and interface along the X_3-axis are identically equal to one. In Fig. 1, we denote by D_0, D_1, Q and d_0, d_1, q, the cross sections (cut by the X_1–X_2 or x_1–x_2 planes, respectively) of the matrix, inhomogeneity and interface in the undeformed and deformed configurations, respectively. In particular, in what follows, the superscripts (0), (1) and (i) are used to identify the corresponding quantities in the matrix, inhomogeneity and interfacial regions, respectively, while the terms 'in-plane' or 'out-of-plane' are used to distinguish quantities in or out of the X_1–X_2 (or x_1–x_2) plane.

To determine the geometric parameters of the inhomogeneity for its neutrality, we should first derive the constitutive equations for the interface membrane. Adopting the continuum framework for hyperelastic materials, the second Piola-Kirchhoff interface stress tensor $\boldsymbol{\Sigma}^{(i)}$ and the Cauchy interface stress tensor $\boldsymbol{\sigma}^{(i)}$ can be given in terms of the interface strain energy density $W^{(i)}$ (measured in the undeformed configuration) and the interface deformation gradient tensor $\boldsymbol{F}^{(i)}$ as

$$\boldsymbol{\Sigma}^{(i)} = 2\frac{\partial W^{(i)}}{\partial \boldsymbol{C}^{(i)}},\quad \boldsymbol{C}^{(i)} = (\boldsymbol{F}^{(i)})^{\mathrm{T}}\boldsymbol{F}^{(i)}, \tag{1}$$



(a) undeformed configuration (b) deformed configuration

Fig. 1. Hyperelastic inhomogeneity-matrix system under finite plane deformations.

$$\boldsymbol{\sigma}^{(i)} = \frac{1}{\det \boldsymbol{F}^{(i)}}\boldsymbol{F}^{(i)}\boldsymbol{\Sigma}^{(i)}(\boldsymbol{F}^{(i)})^{\mathrm{T}}. \tag{2}$$

Here, since the interface is modeled as a membrane (vanishing thickness, stretching only and no bending resistance), each of the interface tensors appearing in Eqs. (1) and (2) is two-dimensional. Introducing the isotropic strain energy density $W^{(i)}$ in the interface as

$$W^{(i)} = W^{(i)}(J_{\mathrm{I}}, J_{\mathrm{II}}),\ J_{\mathrm{I}} = (\lambda_{\mathrm{I}}^{(i)})^2 + (\lambda_{\mathrm{II}}^{(i)})^2,\ J_{\mathrm{II}} = (\lambda_{\mathrm{I}}^{(i)})^2(\lambda_{\mathrm{II}}^{(i)})^2, \tag{3}$$

where J_{I} and J_{II} are the two invariants of $\boldsymbol{C}^{(i)}$ with $\lambda_{\mathrm{I}}^{(i)}$ and $\lambda_{\mathrm{II}}^{(i)}$ representing the two principal stretches, then using the well-known results (here $\boldsymbol{I}^{(i)}$ denotes the two-dimensional interface identity tensor defined in the tangent plane to the interface)

$$\frac{\partial J_{\mathrm{I}}}{\partial \boldsymbol{C}^{(i)}} = \boldsymbol{I}^{(i)},\ \frac{\partial J_{\mathrm{II}}}{\partial \boldsymbol{C}^{(i)}} = J_{\mathrm{II}}(\boldsymbol{C}^{(i)})^{-1}, \tag{4}$$

Eq. (1) becomes

$$\boldsymbol{\Sigma}^{(i)} = 2\left[\frac{\partial W^{(i)}}{\partial J_{\mathrm{I}}}\boldsymbol{I}^{(i)} + J_{\mathrm{II}}\frac{\partial W^{(i)}}{\partial J_{\mathrm{II}}}(\boldsymbol{C}^{(i)})^{-1}\right]. \tag{5}$$

Substituting Eq. (5) into Eq. (2) and noting $\det \boldsymbol{F}^{(i)} = \sqrt{J_{\mathrm{II}}}$ we obtain

$$\boldsymbol{\sigma}^{(i)} = \frac{2}{\sqrt{J_{\mathrm{II}}}}\left[\frac{\partial W^{(i)}}{\partial J_{\mathrm{I}}}\boldsymbol{F}^{(i)}(\boldsymbol{F}^{(i)})^{\mathrm{T}} + J_{\mathrm{II}}\frac{\partial W^{(i)}}{\partial J_{\mathrm{II}}}\boldsymbol{I}^{(i)}\right], \tag{6}$$

from which the principal Cauchy stresses $\sigma_{\mathrm{I}}^{(i)}$and $\sigma_{\mathrm{II}}^{(i)}$ are given by

$$\sigma_{\mathrm{I}}^{(i)} = 2\lambda_{\mathrm{I}}^{(i)}\left[\frac{\partial W^{(i)}}{\partial J_{\mathrm{I}}}\frac{1}{\lambda_{\mathrm{II}}^{(i)}} + \frac{\partial W^{(i)}}{\partial J_{\mathrm{II}}}\lambda_{\mathrm{II}}^{(i)}\right],\ \sigma_{\mathrm{II}}^{(i)} = 2\lambda_{\mathrm{II}}^{(i)}\left[\frac{\partial W^{(i)}}{\partial J_{\mathrm{I}}}\frac{1}{\lambda_{\mathrm{I}}^{(i)}} + \frac{\partial W^{(i)}}{\partial J_{\mathrm{II}}}\lambda_{\mathrm{I}}^{(i)}\right]. \tag{7}$$

In particular, for the specific problem considered here (plane strain deformations), $\lambda_{\mathrm{I}}^{(i)}$ denotes the in-plane stretch along the in-plane tangent to the interface determined by the ratio of the arc length of two corresponding infinitesimal elements on the curves q and Q, while $\lambda_{\mathrm{II}}^{(i)}$ refers to the out-of-plane stretch along the X_3-axis and is therefore always identically equal to one. Consequently, the normal Cauchy stress $\sigma_{tt}^{(i)}$ (along the tangent t to the curve q) in the interface membrane is exactly the principal Cauchy stress $\sigma_{\mathrm{I}}^{(i)}$ and therefore can be given by

$$\sigma_{tt}^{(i)} = 2\lambda_{\mathrm{I}}^{(i)}\left(\frac{\partial W^{(i)}}{\partial J_{\mathrm{I}}} + \frac{\partial W^{(i)}}{\partial J_{\mathrm{II}}}\right)\Bigg|_{J_{\mathrm{I}}=(\lambda_{\mathrm{I}}^{(i)})^2+1,\,J_{\mathrm{II}}=(\lambda_{\mathrm{I}}^{(i)})^2}, \tag{8}$$

the square of which is

$$(\sigma_{tt}^{(i)})^2 = 4(\lambda_{\mathrm{I}}^{(i)})^2\left(\frac{\partial W^{(i)}}{\partial J_{\mathrm{I}}} + \frac{\partial W^{(i)}}{\partial J_{\mathrm{II}}}\right)^2\Bigg|_{J_{\mathrm{I}}=(\lambda_{\mathrm{I}}^{(i)})^2+1,\,J_{\mathrm{II}}=(\lambda_{\mathrm{I}}^{(i)})^2}. \tag{9}$$

We note that the right side of Eq. (9) can be treated as a function of the quantity $(\lambda_{\mathrm{I}}^{(i)})^2$ and usually takes the following polynomial form [19,21]:

$$(\sigma_{tt}^{(i)})^2 = \sum_{j=0}^{N}\gamma_j[(\lambda_{\mathrm{I}}^{(i)})^2 - 1]^j, \tag{10}$$

where γ_j (j=1...N) are a series of constants determined by the strain energy density $W^{(i)}$. In fact, Eq. (10) can be also treated as a truncated Taylor series in $(\sigma_{tt}^{(i)})^2$ with respect to $(\lambda_{\mathrm{I}}^{(i)})^2$so that even when a specific strain energy density $W^{(i)}$ does not immediately yield $(\sigma_{tt}^{(i)})^2$ in polynomial form, Eq. (10) can still be used without inducing large errors given N is taken to be sufficiently large. Consequently, effectively, Eq. (10) is applicable for an arbitrary interface represented by an isotropic (hyperelastic) membrane. In what follows, we examine the neutrality of the inhomogeneity with interface effect based on Eq. (10).

3. Analysis

For a neutral inhomogeneity, the Cauchy stress field inside the matrix is uniform when a uniform external loading is imposed on the

Download English Version:

https://daneshyari.com/en/article/5016587

Download Persian Version:

https://daneshyari.com/article/5016587

Daneshyari.com