International Journal of Non–Linear Mechanics 88 (2017) 11–20

Contents lists available at ScienceDirect

International Journal of Non–Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Interaction between snap-through and Eulerian instability in shallow structures

F. Bazzucchi*, A. Manuello, A. Carpinteri

Politecnico di Torino, Department of Structural, Geotechnical and Building Engineering, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

ARTICLE INFO

Keywords:
Structural stability
Snap-through
Buckling
Nonlinear analysis
Interaction analysis
Equilibrium path
Imperfection sensitivity
Coupled instability

ABSTRACT

The multifold nature of structural instability problems necessitates a number of different kinds of analytical and numerical approaches. Furthermore, instability collapses of large-span roof sensitized the global community to reduce the effects of geometrical imperfections, then some limiting recommendations have been recently proposed. This study provides new insights into the interaction between the two different categories of structural instability and, for the first time, a unified theoretical evaluation of the critical load due to interaction is proposed. The snap-through phenomenon of 2D Von Mises arches was investigated by an incremental-displacement nonlinear analysis. At the same time, the equilibrium paths were considered in relation to the Eulerian buckling loads for the same structural systems. For each structural scheme the effect of the two governing parameters was investigated: slenderness and shallowness ratios. For these purposes, several original theoretical and numerical snap-through versus buckling interaction curves were obtained. These curves provide indications about the prevailing collapse mechanism with regards to the geometric configuration of the structure. Consequently, this innovative method is able to predict the actual instability of a wide range of mechanical systems. With this approach, it is possible also to establish the connection between the magnitude of structural imperfections (defects) and instability behavior. The proposed procedure is able to provide the effective critical load given by the interaction effect and to correlate the instability behavior to the maximum tolerable imperfection sizes.

1. Introduction

Traditionally, in Civil Engineering, Eulerian buckling and snap-through instabilities are treated separately because they are supposed to take place with different modalities and physical mechanisms. However, such phenomena can interact in real structures implying more critical scenarios (*interaction*) than the ones obtained by separated analyses. One of the main consequences is that the greater the interaction the greater the imperfection sensitivity. A typical case where the interaction effect is appreciable is that of shallow structures – arches or domes – in which the presence and the magnitude of the interaction mainly depend on the slenderness and the shallowness ratios.

It has long been recognized that slender structures generally loses their stability by “snapping” or “buckling”. On one hand, from a theoretical point of view, the structure is said to snap when the equilibrium path, emerging from the unloaded state, loses its stability on yielding the first locally maximum value of the load. Commonly, this condition occurs when the structure works mostly through a compressive regime of internal forces. On the other hand, the theoretical critical condition is reached when the structure is said to buckle and the equilibrium path loses its stability at a point of bifurcation. The study of the latter condition reached a very complete treatment in the work of Koiter [1], who provided the fundamentals to determine the imperfection sensitivity of a structural system by the investigation of the pre-critical elastic buckling condition. This study represents the first step of a more refined and generalized theory on the elastic buckling. A theory that is rooted in the works of some of the most eminent scientists of the Eighteenth Century [2,3]. On the contrary, the snap-through instability, even if widely accepted, did not find rigorous mathematical collocation until the Sixties. Concerning this topic, in fact, the main contribution has been given by Thompson [4,5] where the analytical conditions in which snap-trough may occur are enounced theoretically. This author, founding his work on the expression of the total potential energy by series expansion, demonstrated that the limit point condition (*snap-through*) is related to a zero value of one of the coefficients of the expansion and, if two null coefficients are present, the bifurcation point is recognized. It is worth to note that this bifurcation occurs in a non-

* Corresponding author.
E-mail address: fabio.bazzucchi@polito.it (F. Bazzucchi).

http://dx.doi.org/10.1016/j.ijnonlinmec.2016.10.006
Received 8 April 2016; Received in revised form 12 September 2016; Accepted 16 October 2016
Available online 18 October 2016
0020-7462/

linear regime, because the pre-critical deformations are not neglected. Thompson's theories find the natural prosecution in the works of Huseyin [6,7], who extended the definition of the instability conditions to the case of multiple independent loads. In order to do this, Huseyin used the definition of stability domains, because the nonlinear essence of the problems makes ineffective the use of superposition principle to obtain the equilibrium path of the structure. The obtained surface (a curve when only two independent parameters are considered) is the boundary over which the critical condition occurs. Working on these parameters, the nature of the instability (snap-through or buckling) cannot be recognized a priori, thus additional conditions must be defined aside the total potential energy definition [8,9].

These studies represent the fundamentals of a general theory on elastic stability and they have a remarkable importance since allow to handle the real systems (affected by imperfections) by the simple definition of a pre- deformed geometrical configuration in the Lagrangian equations. An appropriate example can be found in the work of Pecknold et al. [11].

In the last two decades, a consistent effort in how to evaluate the imperfection sensitivity of shallow and slender structures has been carried out by the scientific community. Operating this way, instability problems become primary and many optimization procedures (based on instability conditions) have been proposed. Among these studies, the coincidence of the critical loads related to different buckling modes has been proposed (*Bleich-Shanley postulate*) [10]. At the same time, the studies by Koiter, Thompson, and Huseyin demonstrated that interaction of different equilibrium paths could lead the system to an higher imperfection sensitivity. Finally, Gioncu [12] observed that in the real structures the presence of imperfections implies that the intersection between two equilibrium paths degenerates to a limit point. The greater the imperfection, the smaller the consequential critical load [12].

To this day, the study of snap-through of framed structures is carried out mostly in the field of deployable [13] and shape-changing systems [14], because new stable spatial configurations can be achieved by the occurrence of a snap instability. The so-called *buckling induced* systems have been recently proposed for MEMS devices [15], and they are based on the same concepts of shape-changing structures. For all these applications, the control of the displacements and (above all) of the actual maximum instability load is essential. Despite the fact that original researches about this topic are not so extended, some works have proposed new approaches on the study of the Von Mises truss [16] and some original derivations of it [17] to evaluate the actual displacements and instability loads.

In this paper, the investigation on the interaction between the two different categories of structural instabilities was presented and, for the first time, a unified theoretical evaluation of the critical load due to interaction was proposed. Moreover, the theories of Gioncu were confirmed even for the case of the interaction between the two different typologies of instability, snap-through and buckling. On the other hand, results demonstrated how the magnitude of the imperfections is not very significant with this type of interaction. To this purpose, the snap-through phenomenon of 2D Von Mises arches was investigated by an incremental-displacement nonlinear analysis. At the same time, the equilibrium paths were considered in relation to the Eulerian buckling loads for the same structural systems.

For each structural scheme, different parameters were analyzed, such as slenderness, together with arch depth. Several original theoretical and numerical snap-through versus buckling interaction curves were obtained. These curves provide indications about the prevailing collapse mechanism with regards to the geometric configuration of the structure. Consequently, this innovative method is able to predict the actual instability of a wide range of mechanical systems. With this approach, it is also possible to establish the relationship between the magnitude of the structural imperfections (defect) and instability behavior. The proposed procedure is able to provide the

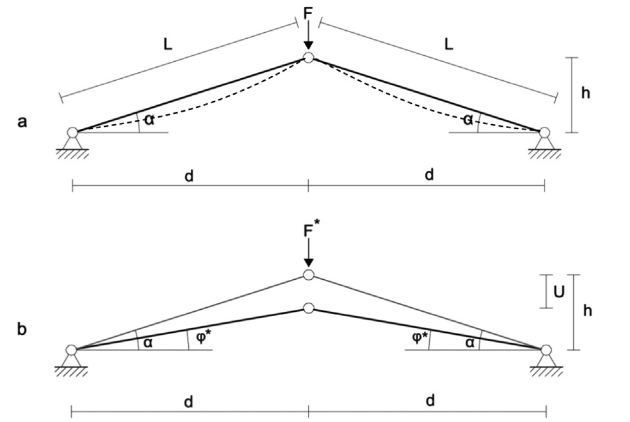

Fig. 1. a) Eulerian elastic buckling of a Von Mises Arch and the adjacent configuration (dotted lines). **b)** The generic equilibrium configuration along a displacement control analysis, identified by F^* and φ^*.

effective critical load given by the interaction effect and to correlate the instability behavior to the maximum tolerable imperfections. Therefore, this study makes a major contribution to research on non-linear instability of structures by demonstrating that the evaluation of the critical load can be affected by the interaction of different-order phenomena.

2. Interaction between elastic buckling and snap-through instability

2.1. Interaction in a simple structure

The mechanical system represented in Fig. 1a is traditionally called Von Mises Arch [18]. This simple structure is made of two hinged bars subjected to a vertical force F applied at the crown. As is well-known, the arch has a nonlinear pre-critical behavior, which involves a snap-through if the equilibrium path is followed beyond the limit point. This same system can be analyzed in respect to the elastic buckling, assuming a non-deformed pre-critical configuration. For a given external load F, the bars react with the axial forces:

$$N = \frac{1}{2}\frac{F}{\sin\alpha}. \tag{1}$$

When N reaches the critical Eulerian buckling load of a hinged bar in compression ($N_{CR,EB}$), the bars lose their flexural stiffness and the equilibrium is established in the adjacent configuration (see Fig. 1a). Under these conditions, the external load is equal to

$$\frac{1}{2}\frac{F}{\sin\alpha} = N_{CR,EB} = \pi^2\frac{EI}{L^2}, \tag{2}$$

$$F_{CR,EB} = 2\pi^2\frac{EI}{L^2}\sin\alpha. \tag{3}$$

In order to determine the more burdensome instability condition, this load must be compared to the snap-through instability load [4,19]:

$$F = 2EA\sin\varphi\left(1 - \frac{\cos\alpha}{\cos\varphi}\right), \tag{4}$$

in which the value of the critical angle φ_{CR} has to be substituted. This value was obtained by the functional analysis of the equilibrium path. Between the two related limit points, an unstable branch of the path (as indicated by the relative negative slope in Fig. 2) is identified, and the values of the critical angle are

$$\varphi_{1,2} = \pm\arccos\sqrt[3]{\cos\alpha}\,. \tag{5}$$

The limit points are obtained by findings the turning points of Eq. (4), since its zeros represent the equilibrium coordinate of the system.