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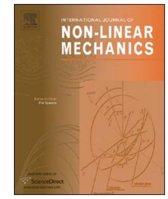

Alternating chaos versus synchronized vibrations of interacting plate with beams

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ABSTRACT

We study networks of coupled oscillators governed by ODEs and yielded by physically validated sets of a few PDEs governing dynamics of structural members (plate and beams), chaos and phase synchronization and contact/no-contact non-linear dynamics of structural members coupled via boundary conditions. We have detected, illustrated and discussed a few novel kinds of hybrid states of the studied plate-beam(s) contact/no-contact interactions as well as novel scenarios of transition into chaos exhibited by the interplay of continuous objects. Classical (time histories, phase portraits, Poincaré maps, FFT, Lyapunov exponents) and non-classical (2D Morlet wavelets) approaches are used while monitoring non-linear dynamics of the interacting spatial structural members. Our results include examples from structural mechanics and the studied objects are modelled by validated mechanical hypotheses and assumptions. Novel non-linear phenomena including switching to different vibration regimes and phase chaotic synchronization are illustrated and discussed.

1. Introduction

There are numerous applications in civil and mechanical engineering regarding structures consisting of linked systems of elastic bodies (beams and plates) with constraints. There is also a large amount of papers and books devoted to modelling of structural members, but surprisingly there is rather a small amount of published research aimed at mathematical modelling of interacting spatial members putting emphasis on their bifurcational and chaotic dynamics. In what follows we briefly review the literature associated with our study, though, our investigation reported in this paper extends our previous studies devoted to chaotic dynamics of structural slender members putting emphasis on interplay of the plate-beams systems.

Moon and Shaw [1] studied chaotic vibrations of a beam with non-linear boundary condition. Then Shaw [2], based on a single mode approximation, analyzed regular and chaotic dynamics as well as subharmonic resonances of an elastic beam with one-sided amplitude constraint subjected to periodic excitation. Non-smooth modelling method has been applied to study non-linear dynamics of a flexible impacting beam by Wagg [3]. An N-degree-of-freedom modal model has been derived using Galerkin's reduction and the numerical simulations have been compared with experimentally recorded data for a one-sided constrained beam. Applicability of simplified models of continuous mechanical systems using example of a conveyor belt has been discussed in reference [4]. Trindade et al. [5] carried out a study of the vibrations of a vertical slender beam, clamped at its upper extreme, pinned in its lower one and constrained inside an outer cylinder in its lower portion. Non-linear stress-strain relations have been taken into account while applying a non-linear finite-element model and using the Karhunen-Loève decomposition. Dumont and Paoli [6] proposed mathematical models governing dynamics of an elastic beam clamped at its left end and located between two rigid obstacles. The studied infinite dimensional contact problem of fully discretized approximations and their convergence have been proved. Seifield [7] studied the transverse impact of steel spheres on aluminium beams numerically and experimentally. Modally-reduced models have been applied in combinations with local finite element contact models and the numerical simulation results have been validated by experimental investigation using Laser-Doppler-Vibrometer. Transient response of a cantilever beam driven by a periodic force and repeated impacting against a rod-like stop have been analyzed by Yin et al. [8]. In both impact and separation phases, the transient wave propagations have been solved using the expansion of transient wave functions. Several transient phenomena have been illustrated and discussed

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including impact-induced waves, sub-impact phases, long term impact motion, chatter, sticking motion, synchronous and non-synchronous impact and impact loss. Ervin and Wickert [9] investigated the forced response dynamics of a clamped-clamped beam with an attached rigid body. Periodic responses, period-doubling bifurcations, grazing impacts, sub-harmonic regions, fractional harmonic resonances as well as chaotic responses have been reported. The finite element method has been employed to study an impacting plate-beam system by Labuschagne et al. [10], when the Reissner-Mindlin plate model combined with the Timoshenko beam model has been used. The paper has been aimed at exhibition of the interface conditions at the contact between the plate and beams and the impact of regularity on the enforcement of certain interface conditions. Ervin [11] studied the repetitive impact dynamics of two orthogonal pinned-pinned beams subjected to base excitation in an analytical way at specified frequency and acceleration. The vibrational process has been monitored in a piecewise fashion as switching between the linear in-contact and not-in-contact states and compatibility conditions have been introduced at their junctions. The studied parameters have been performed on contact stiffness, relative beam stiffness, contact location, modal damping, and stand-off gap. Wang et al. [12] analyzed the resonance characteristic of a two-span continuous beam traversed by moving high speed trains at a constant velocity where each span is modelled as a continuous Bernoulli-Euler beam and the moving trains are represented as a series of two-degrees-of-freedom mass-spring-damper systems. Ryu et al. [13] developed the beam structure models with impact or contact parts under impact forces. Both theoretical and experimental investigations for the dynamics relationships between the impacting device and impact mechanism have been utilized. Hu and Ervin [14] studied Euler-Bernoulli beam with adjustable boundary conditions and variable contact location numerically under a pulse loading. Numerous test cases have been employed with varied pulse duration, pulse amplitude and clearance, and the computational results have been compared with experimental investigations. Yang et al. [15] carried out an analytical study on chaotic characteristics of viscoelastic beams based on the evolution of non-linear stiffness. Quasi-periodic and chaotic responses associated with period doubling bifurcations have been detected and discussed. Geiyer and Kauffman [16] studied linear cantilevered, piezoelectric energy harvesters (piezomagnetic beams) using benefits of chaotic phenomena by stabilizing high energy periodic orbits located within a chaotic attractor. The authors applied a single time series to compute orbit selection, local linearization and control perturbation.

The carried out analysis of the state-of-the art of research devoted to interaction nonlinear dynamics of contacting structural members shows that there is a lack of a study of bifurcational and chaotic dynamics of a few movable continuous mechanical objects being either in contact or non-contact while vibrating, i.e. exhibiting the so called *switching type non-linear dynamics*.

The literature review shows that mainly contact dynamics of a beam/plate with rigid obstacles have been investigated. There are only a few papers dealing with the mentioned switching type non-linear dynamics of two interacting beams, and mainly the so called reduced-order models have been derived possessing their own drawbacks with respect to neglection of the multi-modes interaction.

It is also rather difficult to find papers dealing with interaction of non-homogeneous continuous members represented for instance by a plate and a series of interacting beams, which is a subject of this paper investigation.

In spite of the physical features of the paper, we offer also important hints to the Mechanics and Mechatronics. The derived PDEs are yielded by modelling of a gyroscopic device, where a contact interaction of a plate and transversal set of beams plays a crucial role in the gyroscope working regimes. Furthermore, the studied in this paper slender structures have a wide application in different navigation devices, electronic techniques and in particular in gyroscopes serving as flat, multi-layer micromechanical accelerometers.

Structural systems consisting of beams and a plate with small clearances between them are studied, and the action of a load subjected to an upper layer yield (plate) interaction between the layers and the phase chaotic synchronization is analyzed. On each time step the contact problem is solved while the structural members are coupled only through the boundary conditions. Different cases of interactions of one plate and one, two, or three beams with small clearances are analyzed deeply, among other.

The reported research extends our earlier studies devoted to chaotic and synchronized dynamics of multi-layer beams [17,18].

The paper is organized in the following way. In Section 2 the governing equations are presented including a switching function exhibiting the contact/no-contact non-linear dynamics. Sections 3 and 4 is devoted to study of chaotic and synchronized state of one plate and one (two) beam(s). Section 5 deals with chaotic and switching type dynamics of the plate and three beams. The last Section 6 reports concluding remarks.

2. The governing equations

In this work we study chaotic vibrations and contact interactions of a plate supported by one or a few beams. The upper plate is governed by the Germain-Lagrange PDE, where beam vibrations are described by the Euler-Bernoulli equations. We assume that the plate and beams are isotropic, a clearance between them is small, and that vibrations of the structural elements are uncoupled and they are governed only by the separated boundary conditions (a contact between structural members yields action of all boundary conditions). The following PDEs govern the plate-beams system dynamics:

$$\frac{1}{12(1-\mu^2)}\nabla_\lambda^4 + \frac{\partial^2 w_1}{\partial t^2} + \varepsilon\frac{\partial w_1}{\partial t} - q_1(x, y,$$
$$t) + P_{1x}\frac{\partial^2 w_1}{\partial x^2} + P_{1y}\frac{\partial^2 w_1}{\partial y^2} - \sum_{i=2}^{n} K(w_1 - w_i - h_k)\Psi_{i-1} = 0,$$
$$\frac{1}{12}\frac{\partial^4 w_i}{\partial x^4} + \frac{\partial^2 w_i}{\partial t^2} + \varepsilon\frac{\partial w_i}{\partial t} - q_i(x, t) + P_i\frac{\partial^2 w_i}{\partial x^2} + K(w_1 - w_i - h_k)\Psi_{i-1} = 0,$$
$$\nabla_\lambda{}^4 = \frac{1}{\lambda^2}\frac{\partial^4}{\partial x^4} + \lambda^2\frac{\partial^4}{\partial y^4} + 2\frac{\partial^4}{\partial x^2\partial y^2}, \quad (1)$$

where the switching function is $\psi_{i-1} = \frac{1}{2}[1 + sign(w_1 - h_k - w_i)]$. Relation $K(w_1 - w_i - h_k)\Psi_{i-1}$ models a contact pressure between the layers, whereas the contact interaction between plates and beams is governed by the Winkler model. We take $q_i(x, t) = 0$. Observe that $\Psi_{i-1} = 1$ if $w_1 > w_i + h_k$, then a contact between plate and beam occurs, otherwise $\Psi_{i-1} = 0$; w_1, w_i are the functions of deflections of the plate and beams, respectively; K is the stiffness coefficient of the transversal structural coupling in a contact zone, and h_k stands for clearances between the structural members.

Eq. (1) are recast to their counterpart non-dimensional form via the following relations: $x = a\bar{x}$, $y = b\bar{y}$; $q = \bar{q}\frac{E(2h)^4}{a^2b^2}$, $\tau = \frac{ab}{2h}\sqrt{\frac{\gamma}{Eg}}$, $\lambda = \frac{a}{b}$, where a, b are the plate dimensions regarding x, y, respectively; t – time, ε – damping coefficient, w – deflection,$2h$ – plate thickness, $\mu = 0.3$ – Poisson's coefficient, g – acceleration of gravity, E – Young modulus, $q_1(x, y, t)$ – transverse load acting on the plate, $q_i(x, t)$ – transverse loads acting on a beam, $P_{1x}(y, t)$, $P_{1y}(x, t)$– longitudinal loads acting on the plate, $P_i(t)$– longitudinal loads acting on the beams, γ/g– unit mass density of the structural material. Bars over non-dimensional parameters in Eq. (1) are omitted.

The governing equations are supplemented by the boundary and initial conditions. In addition, a condition of a lack of penetration between contacting flexible structural members should be added. The obtained set of non-linear PDEs (1) is reduced to a counterpart set of the second-order ODEs using the Bubnov-Galerkin approach. Functions w_1 and w_2 are the solutions of Eq. (1) and they are approximated by the following series