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Geometrically exact curved beam element using internal force field defined in deformed configuration

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ABSTRACT

This paper presents a study on the development of high-performance finite elements for geometrically nonlinear analysis of frame structures with curved members. Based on the geometrically exact beam theory, a highly efficient and accurate mixed finite element is developed. A new approach is proposed for constructing the independent internal force field by including major terms satisfying equilibrium conditions in the deformed configuration. An element-level equilibrium iteration procedure is employed for the condensation of element internal degrees of freedom during the nonlinear solution. Numerical results are presented to demonstrate the excellent performance of the element developed, and it is shown that even when each structural member is modelled with just one element, accurate solutions can still be achieved.

1. Introduction

While much research effort has been devoted to nonlinear analysis of frame structures over the years, it may still be difficult to accurately predict nonlinear behaviours of spatial frame structures. This difficulty is partly due to the fact that a curved geometry of the structural member may cause significant couplings between extension, bending, and torsion, and such couplings are complicated to simulate. Thus, it remains a challenge to develop accurate and efficient geometrically nonlinear curved beam elements for spatial frame structures.

Considerable progress has been made on geometrically nonlinear analysis of spatial frame structures in the last several decades. Among the most important developments, the geometrically exact beam theory of Reissner [1] and Simo [2] has been widely used. Geometrically exact beam is a type of nonlinear beam model based on finite rotation theory. The spatial state of a beam is described by a vector defining the centroid position of each cross-section, and a rotation tensor defining the orientation of the cross-section. The effect of transverse shear deformation can be included because the cross-sections are no longer assumed normal to the deformed line of centroids. Based on this beam model, many studies have been carried out, e.g. Simo and Vu-Quoc [3], Cardona and Geradin [4], Pimenta and Yojo [5], Ibrahimbegović et al. [6], Ibrahimbegović [7], Ritto-Corrêa and Camotim [8], Zupan et al. [9], Pai [10], Češarek et al. [11], Gacesa and Jelenic [12], Zhong et al. [13], Meier et al. [14], Mueller et al. [15], Xiao and Zhong [16], Zhang and Zhong [17]. Applications of the theory have also been extended to the analysis of initially curved beams, e.g. Ibrahimbegović [18], Zupan and Saje [19], Kapania and Li [20], Mata et al. [21], Meier et al. [22].

Parameterization of the rotation field is an essential issue in the implementation of geometrically exact beam. As spatial rotations belong to a nonlinear manifold, improper parameterization of spatial rotations may make the computation cumbersome and induce inaccuracy [13]. There are several alternative parameterization methods available, including Euler parameters, rotational quaternion, and the rotation vector [7–9,23]. Among them, the rotation vector method is regarded as very promising. One of its major advantages is that the nodal rotation vectors can be additively updated and the curvature can be computed directly from the rotation vector variation along the beam. Another advantage is that there is no accumulated error from the configuration updating with the total values of rotation vector, making it possible to get solutions insensitive to load increments. Based on the method proposed by Cardona and Geradin [4], Pimenta and Yojo [5] derived a symmetric tangent stiffness matrix of conservative system from linearized weak forms of the equilibrium equation. Ibrahimbegović [7] and Ibrahimbegović et al. [6] further proposed the computational method for the singularity problem. Ritto-Corrêa and Camotim [8] presented explicit expressions for differentiating the

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Rodrigues formula and the spin-rotation vector variation relationship up to the second directional derivative. Since then, the effective rotation vector parameterization has been established.

It is well known that, solution accuracy of displacement-based elements is relatively low, especially for stresses and internal forces. In order to get more accurate solutions, more elements are usually required and computational efforts will greatly increase. Another issue with displacement-based elements is the so-called locking phenomena. Shear and membrane locking are serious problems associated with displacement-based beam elements with independent displacement and rotational fields and/or curved geometry [13,24]. The use of mixed finite element models is one of the effective approaches to overcome such numerical problems. Saleeb and Chang [25] showed that the membrane and shear locking phenomena of relatively thin curved beams can be eliminated by using mixed finite elements with appropriate conditions. Studies on ordinary beam by Neuenhofer and Filippou [26], Hjelmstad and Taciroglu [27], Sun and Bursi [28], Tort and Hajjar [29] and Alemdar and White [30] illustrated that mixed finite elements have advantages on computational accuracy and efficiency over conventional displacement-based elements. For geometrically exact beams, a few studies have been reported, including [31–34]. Nukala and White [31] developed a geometrically exact mixed thin-walled beam element ignoring the effect of transverse shear deformation. Santos et al. [32] developed a two-field hybrid-mixed beam element based on geometrically exact beam theory and their numerical examples showed that the mixed finite element did not suffer from shear locking, but it was limited to linear transversal displacement fields and hence the effect of bending can't be accurately considered. Wackerfuß and Gruttmann [34] gave a specific formulation of the three-field mixed beam element, however the effect of element spatial configuration on definition of internal force fields was not included. So far, no mixed curved beam element has been reported which can consider the nonlinear effects of finite rotation, transverse shear deformation and element spatial configuration simultaneously.

In the present study, a mixed curved beam element is developed for accurate and efficient nonlinear analysis of general frame structures, with effects of finite rotation, shear deformation, and spatial configuration all considered. The element formulation is based on the geometrically exact beam theory and the Hellinger-Reissner variational principle. Lagrange interpolation is used for translational displacement and rotation fields, and beam rotations are expressed with the rotation vector. A new approach is proposed and employed for constructing the independent internal force field by considering the equilibrium relationship at deformed configuration. An element-level equilibrium iteration procedure is employed for condensing out some of the nodal unknowns that are treated as internal DOFs of the element.

2. Geometrically exact curved beam theory

2.1. Kinematic description

Geometry of a three-dimensional beam is described by the family of plane cross-sections and the line of centroids of cross-sections. Each of the cross-sections is assumed to remain planar and preserve its initial shape, but is not necessarily normal to the line of centroids at deformed state. This makes it possible to include the effect of shear deformation.

For the sake of simplicity, only beams curved in a single plane is considered in this study. In the Cartesian reference system with base vectors $\mathbf{E}_i$ $(i = 1, 2, 3)$, two configurations are defined for a beam element initially curved in plane 1–2 (see Fig. 1):

(i) The initial configuration — the curved beam configuration at the initial state, in which mechanical properties of the beam are defined and loading conditions are given;
(ii) The current configuration — the beam configuration at a deformed state.

The geometry of the initial configuration is invariable and described by the family of position vectors $\mathbf{r}_0 = [r_{01} \;\; r_{02} \;\; r_{03}]^{\mathrm{T}}$ of the line of centroids and the orthonormal base vectors $\mathbf{e}_{0i}$ $(i = 1, 2, 3)$. The current configuration is described by the family of position vectors $\mathbf{r} = [r_1 \;\; r_2 \;\; r_3]^{\mathrm{T}}$ and the orthonormal base vectors $\mathbf{e}_i$ $(i = 1, 2, 3)$. Vectors $\mathbf{r}_0$, $\mathbf{e}_{01}$, $\mathbf{e}_{02}$, $\mathbf{e}_{03}$, $\mathbf{r}$, $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are expressed as functions of S, which is the arc-length parameter. In Fig. 1, A_0 and B_0, which are both in plane 1–2, are the start point and the end point of the initial beam configuration, respectively; A and B are the start point and the end point of the current beam configuration. Note that different base vectors can be related through the following expression

$$\mathbf{e}_i = \boldsymbol{\Lambda}\mathbf{e}_{0i} = \boldsymbol{\Lambda}\boldsymbol{\Lambda}_0\mathbf{E}_i \quad (i = 1, 2, 3) \tag{1}$$

where $\boldsymbol{\Lambda}$ is the rotation tensor positioning the initial configuration onto the current configuration and it can be parameterized by the rotation vector $\boldsymbol{\theta} = [\theta_1 \;\; \theta_2 \;\; \theta_3]^{\mathrm{T}}$, which is a vector aligned with the rotation axis and having a magnitude equal to the rotated angle $\theta = \|\boldsymbol{\theta}\|$, given by Rodrigues formula [8]

$$\boldsymbol{\Lambda} = \mathbf{I}_{3\times3} + \frac{\sin\theta}{\theta}[\boldsymbol{\theta} \times \mathbf{I}_{3\times3}] + \frac{1-\cos\theta}{\theta^2}[\boldsymbol{\theta} \times \mathbf{I}_{3\times3}]^2 \tag{2}$$

where $\mathbf{I}_{3\times3}$ is the 3×3 identity matrix. And also in Eq. (1), $\boldsymbol{\Lambda}_0$ is the rotation tensor positioning the reference basis $\mathbf{E}_i$ onto the initial configuration and is given by

$$\boldsymbol{\Lambda}_0 = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where θ_0 is the angle between basis vectors $\mathbf{e}_{01}$ and $\mathbf{E}_1$.

Based on the rotation vector parameterization, the beam configuration can be completely defined by vectors $\mathbf{r}$ and $\boldsymbol{\theta}$, which can be grouped together as generalized displacements,

$$\boldsymbol{\phi} = \begin{bmatrix} \mathbf{r} \\ \boldsymbol{\theta} \end{bmatrix} \tag{4}$$

By using values of the generalized displacements, path-independent and invariant finite element implementation can be achieved. These advantages make it possible to get solutions insensitive to step sizes in the incremental/iterative solution procedure.

For the sake of simplicity, a rectangular cross-section with width b and height h is considered in the following derivation. Introducing a standard hexahedral element with natural coordinates ξ, η, ζ for the beam axis direction, cross-sectional height and width directions (see Fig. 2), we can express the initial and current configurations in the following form

$$\begin{aligned}\mathbf{X}(\xi, \eta, \zeta) &= [X_1(\xi, \eta, \zeta) \;\; X_2(\xi, \eta, \zeta) \;\; X_3(\xi, \eta, \zeta)]^{\mathrm{T}} \\ &= \mathbf{r}_0(\xi) + \boldsymbol{\Lambda}_0(\xi)\left[\frac{h}{2}\eta\,\mathbf{E}_2 + \frac{b}{2}\zeta\,\mathbf{E}_3\right]\end{aligned} \tag{5}$$

$$\begin{aligned}\mathbf{x}(\xi, \eta, \zeta) &= [x_1(\xi, \eta, \zeta) \;\; x_2(\xi, \eta, \zeta) \;\; x_3(\xi, \eta, \zeta)]^{\mathrm{T}} \\ &= \mathbf{r}(\xi) + \boldsymbol{\Lambda}(\xi)\boldsymbol{\Lambda}_0(\xi)\left[\frac{h}{2}\eta\,\mathbf{E}_2 + \frac{b}{2}\zeta\,\mathbf{E}_3\right]\end{aligned} \tag{6}$$

where $\boldsymbol{\Lambda}_0(\xi)$ is given by Eq. (3) with $\cos\theta_0(\xi)$ and $\sin\theta_0(\xi)$ expressed as

$$\cos\theta_0(\xi) = \frac{1}{J_s}\frac{\mathrm{d}r_{01}(\xi)}{\mathrm{d}\xi}, \quad \sin\theta_0(\xi) = \frac{1}{J_s}\frac{\mathrm{d}r_{02}(\xi)}{\mathrm{d}\xi} \tag{7}$$

with

$$J_s = J_s(\xi) = \sqrt{\left[\frac{\mathrm{d}r_{01}(\xi)}{\mathrm{d}\xi}\right]^2 + \left[\frac{\mathrm{d}r_{02}(\xi)}{\mathrm{d}\xi}\right]^2} = \frac{\mathrm{d}S(\xi)}{\mathrm{d}\xi} \tag{8}$$

The Jacobian matrix for coordinate transformation is