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Similarity solutions for power-law and exponentially stretching plate motion with cross flow

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ABSTRACT

Laminar boundary layers generated by power-law plate stretching with cross flows are studied. Only the stretching solutions of Banks [10] are considered, those being bounded by exponentially stretched plates. In one case the cross flow is generated by a uniform transverse stream far above the stretching plate or a wall moving with uniform transverse velocity. Two other cases deal with cross flows generated by transverse shearing motions of the surface. Possible two parameter solutions appear, but here we present two one-parameter families of cross flow solutions generated by transverse plate shearing motion. Streamwise and transverse shear stresses and velocity profiles are displayed in graphical form.

1. Introduction

The study of cross flows began some time after the pioneering studies by Prandtl [1] and Blasius [2] on the laminar flow over a flat plate with small viscosity. Indeed, Prandtl [3] is apparently the first to report the solution for uniform press gradient flow past an infinite yawed cylinder. Various theoretical developments through the following years have been reviewed by Cooke and Hall [4] and a review of numerical methods for solving generalized three-dimensional boundary layer flows is given by Eichelbrenner [5].

The basic feature of similarity flows of this type is that the *primary* streamwise varying flow is described by a self-contained ordinary differential equation while the *secondary* fully-developed corss-flow is described by a linear ordinary differential equation which has, for its variable coefficients, terms involving the primary flow solution. This one-way coupling has been labeled the 'independence principle' by Jones [6].

Relatively recent studies on cross flows include that of Weidman [7] and Fang and Lee [8]. A very recent study on cross flows induced solely by transverse plate motions has been reported by Weidman [9].

In the current study we consider flows transverse to the flow induced by streamwise power-law stretching surfaces studied by Banks [10]. The cross flows may be generated by a uniform cross flow stream above the plate or by uniform plate motion. Also, similarity solutions for cross flows generated by transverse shearing motions of the surface are available. Here we consider only streamwise stretching plates. This excludes the set of solutions of Banks for which the plate moves upstream.

The presentation is as follows. The theoretical problem developed in Section 2 includes a summary of Banks' power-law stretching plate similarity fomulation. In Section 3 the cross flows obtained for a uniform free stream above the plate or a plate moving transverse at uniform velocity are presented. Cross flows generated by transverse shearing motions of the plate are presented in Section 4. A discussion of results and concluding remarks are given in Section 5. A precise derivation of the equation for exponential stretching of a plate found by Banks is given in the Appendix.

2. Theoretical development

The theory is developed following the notation of Banks [10]. We take Cartesian coordinatews (x, y, z) with associated velocities (u, v, w). The streamwise flow are directed along the x-direction, y is the plate normal coordinate and z is the spanwise coordinate in the direction of the cross flow. All flows considered are of infinite extent in the spanwise direction, and thus are fully-developed. Hence we look for solutions with velocity fields independent of the spanwise coordinate z, so the continuity equation for incompressible flow reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

and the constant property boundary layers equations reduce to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2.2a}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{2.2b}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{2.2c}$$

where ρ is the fluid density and ν is its kinematic viscosity. The first

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viscous term in each equation is neglected in the boundary-layer approximation. Also, since the flows considered have no external pressure gradient, the plate-normal Eq. (2.2b) is neglected. Consequently, the leading order boundary-layer description of the streamwise and cross-flow momentum equations reduce to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2.3a}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \nu\frac{\partial^2 w}{\partial y^2}. \tag{2.3b}$$

2.1. Power-law stretching surfaces

The flow induced by power-law stretching of a plate $u = ax^m$ over a flat plate with leading edge at $x=0$ is described by the similarity velocities [10]

$$u(x, y) = ax^m f'(\eta), \quad \eta(x, y) = \sqrt{\frac{a(m+1)}{2\nu}}\, x^{(m-1)/2}\, y \tag{2.4a}$$

$$v(x, y) = -\sqrt{\frac{a(m+1)}{2\nu}}\, x^{(m-1)/2}\left[f(\eta) + \left(\frac{m-1}{m+1}\right)\eta f'(\eta)\right]. \tag{2.4b}$$

This gives rise to Banks' equation and impermeable plate boundary conditions

$$f''' + ff'' - \beta f'2 = 0, \quad f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0 \tag{2.5}$$

in which $\beta = 2m/(m+1)$. Banks [10] integrated this equations over the large parameter space $-1.9999 \le \beta \le 202$. In these solutions Banks has shown that flows in the range $-2 < \beta \le 2$ has the plate stretching away from the origin, whilst for $2 < \beta < \infty$ the plate shrinks towards the origin. In this study only stretching plate solutions are considered.

The dimensional form of the longitudinal wall shear stress is given as

$$\tau_x = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \rho\sqrt{\frac{a^3\nu(m+1)}{2}}\, x^{(3m-1)/2} f''(0) \tag{2.6}$$

where μ is the absolute viscosity of the fluid.

Solution of these high Reynolds number equations describe the nature of the flow asymptotically far downstream of the origin ($x = 0$) of the streamwise flow. In the following problems we find one-way coupled ordinary differential equations which are numerically solved using a shooting technique with the ODEINT code of Press et al. [11].

3. Uniform free stream and transverse plate motion

The cases for uniform free stream cross flow and uniform transverse plate motion are intimately related. In either case the similarity ansatz is taken as

$$w(x, y) = W_0\, g_1(\eta) \quad \text{(free stream)}, \qquad w(x, y) = W_0\, g_2(\eta) \quad \text{(plate motion)} \tag{3.1}$$

where W_0 is the free stream cross flow far above the plate with similarity variable $g_1(\eta)$ and it is the uniform transverse plate motion with similarity variable $g_2(\eta)$. Inserting these forms along with Banks' velocity ansatz (2.4) into (2.3b) furnishes the coupled boundary-value problems

$$g''_1 + fg'_1 = 0, \quad g_1(0) = 0, \quad g_1(\infty) = 1. \tag{3.2a}$$

and

$$g''_2 + fg'_2 = 0, \quad g_2(0) = 1, \quad g_2(\infty) = 0. \tag{3.2b}$$

In both cases the dimensional form of the transverse wall shear stress is given by

$$\tau_z = \mu\frac{\partial w}{\partial y}\bigg|_{y=0} = \rho W_0\sqrt{\frac{a\nu(m+1)}{2}}\, x^{(m-1)/2} g'_1(0) \tag{3.3}$$

3.1. Special cases for the $g_1(\eta)$ solution

Certain aspects of the $g_1(\eta)$ flow at $\beta = \{1, 0, -1\}$ are now considered. For $\beta = 1$ we note that the streamwise flow is that due to Crane [12] which has an exact solution for $f(\eta)$. Inserting this solution into (3.2) gives the boundary-value problem

$$g''_1 + (1 - e^{-\eta})g'_1 = 0, \quad g_1(0) = 0, \quad g_1(\infty) = 1 \tag{3.4}$$

which has solution

$$g_1(\eta) = \frac{e}{(e-1)}\left[e^{-e^{-\eta}} - \frac{1}{e}\right] \tag{3.5}$$

giving the cross flow wall shear stress parameter

$$g'_1(0) = \frac{1}{(e-1)} \doteq 0.581977. \tag{3.6}$$

A comparison of the Eq. (3.2a) with the $\beta = 0$ equation $f'' + ff'' = 0$ and their boundary conditions shows that

$$g_1(\eta) = 1 - f'(\eta) \tag{3.7}$$

and numerical calculations verify that the cross flow and streamwise wall shear stress parameters are given by

$$g'_1(0) = -f''(0) = 0.627555. \tag{3.8}$$

As pointed out by Banks [10] the exact solution of (2.5) for $\beta = -1$ is

$$f(\eta) = \sqrt{2}\,\tanh\left(\frac{\eta}{\sqrt{2}}\right). \tag{3.9}$$

Inserting this into boundary-value problem (3.2a) and solving gives

$$g_1(\eta) = \tanh\left(\frac{\eta}{\sqrt{2}}\right) \tag{3.10}$$

from which the cross flow wall shear stress parameter is found to be

$$g'_1(0) = \frac{1}{\sqrt{2}} \doteq 0.707107. \tag{3.11}$$

3.2. Connection between the two solutions

Inspection of the governing equations and boundary-values (3.2a) and (3.2b) show that the solutions are related as

$$g_2(\eta) = 1 - g_1(\eta), \quad g'_2(0) = -g'_1(0) \tag{3.12}$$

so one need only solve the boundary-value problem for $g_1(\eta)$ from which the results for $g_2(\eta)$ are deduced.

To check the symmetry relation (3.12), solutions of the coupled Eqs. (3.2a,b) and (2.5) were obtained over the parameter range $-2 < \beta \le 2$. The resulting streamwise wall shear stress parameter $f''(0)$ and cross flow wall shear stress parameters $g'_1(0)$ and $g'_2(0)$ are shown in Fig. 1 as a function of β. Sample streamwise velocity profiles $f'(\eta)$ at selected values of β are shown in Fig. 2 and corresponding cross flow velocity profiles $g_1(\eta)$ and $g_2(\eta)$ are displayed inf Fig. 3.

4. Transverse wall shearing motions

Motivated by the study of Weidman (2016) we seek transverse shearing motions that might generate a cross flow beneath the streamwise power-law plate stretching motions. Here we posit the solution for transverse motions as

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