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Compressive analysis applied to radiation symmetry evaluation and optimization for laser-driven inertial confinement fusion

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ABSTRACT

Having as symmetric a radiation drive as possible is very important for uniformly imploding the centrally located capsule in laser-driven Inertial Confinement Fusion (ICF). Usually, intensive computation is required to analyze and optimize the radiation symmetry in ICF. In this paper, a novel compressive analysis approach is presented to efficiently evaluate and optimize the radiation symmetry. The core idea includes (1) the radiation flux on the capsule for symmetry evaluation is transformed into frequency domain and weighted to obtain a sparse and orthogonal representation, (2) the sparse coefficients reflecting the radiation flux distribution are accurately and efficiently recovered from far less samples on the frequency domain, i.e. $[0, 2\pi) \times [0, \pi]$ through ℓ 1-norm optimization, which greatly improves the efficiency of radiation symmetry evaluation and optimization for the design of physics experiments in the laser-driven ICF, and (3) the sparsity level to recover the sparse coefficients is adaptively determined with a one-dimensional optimization procedure for accurate and efficient compressive analysis. Finally, two examples on current laser facilities are utilized to demonstrate the evaluation accuracy, robustness and computation efficiency of compressive analysis approach.

1. Introduction

Inertial Confinement Fusion (ICF) is a process in which nuclear fusion reactions are initiated by heating and compressing a fuel capsule containing a mixture of *D*euterium and *T*ritium which can be got from seawater. Currently, research on ICF is very active and fruitful since ICF can potentially generate clean, safe, and economic energy without generating pollution. The indirect-laser-driven approach to ICF is believed to have promise [1–3], in this approach, the lasers are directed into a cylindrical cavity around the capsule, and converted into X-rays to radiate and drive the implosion of the capsule. The regions where laser energy deposits remain hotter than indirectly heated areas, and laser entrance and diagnostic holes also introduce additional asymmetries in the radiation flux on the capsule, which will result in asynchronous shock and asymmetric implosion. Hence, such radiation flux asymmetries should be kept at a lower level to get a uniform implosion, e.g. 2% has been reported in [4]. Thus, radiation symmetry evaluation and optimization is very important in the indirect-laser-driven ICF.

The radiation symmetry evaluation is related to the laser–plasma interactions and transport of X-rays from the hohlraum wall to the capsule, which involves the solving of kinetic equations and hydrodynamics equations, which are very complex, and entail certain difficulties. In practice, simple mathematical models such as view-factor codes are used to compute the radiation flux on the capsule, especially for the preliminary design and optimization of thermonuclear target structure and shape [5,6], and play a complementary role to atomic and hydrodynamic codes [7,8].

To evaluate radiation symmetry on the capsule with the view-factor model, we need to facet the surface patches of cylindrical hohlraum into quadrangular or triangular elements, and choose an appropriate analysis model such as equivalent energy model [7–12] to compute the radiation flux on the capsule, which forms non-linear equations of all the discrete elements. Then the radiation symmetry can be evaluated by transforming the radiation flux into the spherical harmonics (*SPH*) domain and calculating the symmetry parameters such as Root Mean Squared Error (RMSE) σ_0 and Legendre non-uniformities (e.g. P_1, P_2, ..., and P_{10} [4]). The radiation flux for any element on the capsule is related to all the elements in the view for the view-factor model. When faceting refinement increases, the evaluation accuracy of radiation symmetry tends to be higher. However, non-linear equations will mushroom greatly, which leads to a great challenge on efficiently finding an appropriate solution for such large-scale equations. To leverage the intensive computation of large scale equations solving, a two-section model is presented [13,14], in which the incident laser

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energy is uniformly distributed onto the discrete elements of cylindrical hohlraum by considering absorbing or losing energy, and only the radiation flux from the hohlraum to the capsule is computed for radiation symmetry evaluation. Such simplification enables a great reduction of intensive computation since there is no requirement on finding a solution for large-scale equations. Nevertheless, the radiation flux of all the elements on the capsule still needs to be computed and involves intensive computation when the discretization resolution is small enough to achieve comparable evaluation accuracy. In addition, the radiation symmetry is usually optimized to achieve a highly uniform radiation on the capsule by finding appropriate values of the target and physical parameters of incident lasers. Thus, such radiation symmetry evaluation needs to be iterated, and the computation time tends to sharply increase when the iteration times are multiplied. Hence, the radiation symmetry evaluation and optimization needs to be accelerated to enable the rapid design and optimization of ICF experiments.

As described above, we need to transform the radiation flux into the frequency domain, i.e., spherical harmonics domain, and then evaluate the radiation symmetry. In such process, we find that the SPH coefficients are very sparse and no more than twenty orders or $21 \times 21 = 441$ of them are larger than zero, which are enough to accurately describe the radiation flux. Since the radiation flux over the spherical harmonics domain is sparse, it means that only a fraction of elements on the capsule may be enough to recover such sparse coefficients. Meanwhile, there are a series of simulation optimization approaches [15] such as gradient-based [16], random search [17], and kriging based response surface [18] to reduce iteration times. This motivates us to find a compressive analysis method so that it can be combined with available simulation based optimization approaches to reduce the computation time for efficient radiation symmetry evaluation and optimization.

Recently, a novel theory named *compressive sampling* is proposed by Candès and Donoho in signal-processing field [19–21], in which a signal can be perfectly recovered from a fraction of samples, in far less than Nyquist sampling rate, by exploiting its sparsity or compressibility. This technique has gained many applications such as Medical Imaging [22], Analog-to-Information Conversion [23], and Computation biology [24], and computer graphics [25]. Since the radiation flux is sparse over the spherical harmonics domain, which means that sparse coefficients can be accurately recovered from a few compressed samples without intensive radiation computation of all the elements. In addition, compressed sampling and radiation recovering for radiation symmetry evaluation are very beneficial to radiation symmetry optimization since there needs repeated radiation flux computation. To efficiently evaluate and optimize the radiation symmetry on the capsule, we need to (1) elaborately choose a sparse and orthogonal basis to represent the radiation flux on the capsule, (2) determine an appropriate sparsity level for an unknown radiation flux and preplan enough samples to efficiently reconstruct the radiation, and (3) efficiently reconstruct sparse coefficients for radiation symmetry evaluation from compressed samples.

In this paper, we present a novel compressive analysis approach for radiation symmetry evaluation and optimization in the laser-driven ICF, which achieves the following:

- The spherical harmonics is selected and weighted to sparsely and orthogonally represent the radiation flux on the capsule inside a cylindrical hohlraum, and only less than 5% elements are required to accurately recover sparse coefficients through ℓ 1-norm minimization (the maximal reconstruction error is no more than 3×10^{-5}), which significantly improves the efficiency of radiation symmetry evaluation.
- The sparsity level S can be adaptively determined for accurate radiation evaluation without any *a-priori* information on radiation distribution by transforming it into a one-dimensional unconstrained optimization problem to find an optimal sparsity S, so that enough number of samples are pre-planned.
- The compressive analysis combined with current simulation optimization approaches can be employed to efficiently and accurately find a near optimal solution for ignition target design in the laser driven ICF experiments (the distance between the optimal point and the true point is no more than 2×10^{-3}).

The remainder of this paper is organized as follows. Section 2 reviews related work on compressive analysis for radiation symmetry evaluation and optimization. Section 3 presents a two-section radiation symmetry evaluation model. Section 4 introduces the mathematic background of compressive analysis. Section 5 gives a compressive analysis framework for radiation symmetry evaluation and optimization. Section 6 validates and analyzes the efficiency of the presented approach. This paper concludes in Section 7.

2. Literature review

Radiation symmetry evaluation and optimization is one of the most important issues in the indirect laser driven ICF [26,27] to achieve a more uniform radiation environment around the capsule, and is always an active research topic in the physics of plasma [7,13]. Usually, the radiated surfaces are first discretized into smaller elements, incident laser energy is then mapped on the elements for radiation computation, and finally, the radiation flux on the capsule is computed and transformed into the spherical harmonics domain to evaluate radiation symmetry. In such process, a lot of elements need to be generated to achieve a prescribed evaluation accuracy, which will result in a large number of non-linear equations and intensive radiation computation [11,12]. Therefore, radiated surfaces, laser-spots and diagnostic holes are often assumed to be rotationally symmetric and simplified to largely reduce the number of elements [11–14]. Typically, only one-dimensional models are built and the efficiency of radiation symmetry evaluation can be improved. However, the evaluation accuracy of radiation symmetry is limited, and the computed radiation flux is usually not consistent with the resulting distribution of ICF experiments. Hence, three-dimensional geometric elements based radiation flux computation has been discussed in [7,9,10]. Nevertheless, when the size of discrete element reduces by half, the number of elements will be doubled, and the resulting radiation flux computation will increase by a factor of 4, which may take more than half an hour for radiation symmetry evaluation, or one day for even single design parameter optimization. Therefore, a new analysis approach is very essential to accelerate the radiation symmetry evaluation and optimization.

In the signal processing field, a new sampling scheme, i.e. *compressive sampling*, has been proposed in [19,20] to capture and represent compressible signals at a rate significantly below the Nyquist sampling rate [21]. Such scheme has been used in medical image processing [22], or A/D signal transformation [23], computing biology [24], and computer graphics [25]. To apply compressive sampling in radiation symmetry evaluation, we need to determine an appropriate sparse representation, sparsity level, enough samples pre-planning, and sparse coefficients reconstruction algorithm. Nevertheless, how to utilize compressive sampling in the radiation symmetry evaluation and optimization is new.

In this paper, we first introduce the background of radiation symmetry evaluation, optimization and compressive analysis, then present a compressive analysis framework for radiation symmetry evaluation and optimization, and finally validate the approach with two examples running on current Shenguang laser facilities [28].

3. Background of compressive analysis for radiation symmetry in the ICF

In this section, radiation symmetry evaluation, optimization, and compressive analysis are discussed below to enable efficient