ARTICLE IN PRESS

International Journal of Plasticity xxx (2017) 1–26

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

A normalized stress invariant-based yield criterion: Modeling and validation

Qi Hu [a], Xifeng Li [a], Xianhong Han [a], Heng Li [b, *], Jun Chen [a, **]

[a] Department of Plasticity Technology, Shanghai Jiao Tong University, Shanghai 200030, China
[b] School of Material Science and Engineering, Northwestern Polytechnical University, Xi'an 710048, China

ARTICLE INFO

Article history:
Received 22 February 2017
Received in revised form 1 September 2017
Accepted 17 September 2017
Available online xxx

Keywords:
Yield criterion
Stress invariants
Convexity
Associated flow rule
Strength differential effect
Lankford coefficients

ABSTRACT

Transforming five different stress invariants-based yield criteria in I-J_2-J_3 framework into η-ξ-$\bar{\sigma}$ framework demonstrates that all of these yield criteria are the product formulation of the equivalent stress and the function of the normalized third invariant ξ and stress triaxiality η. The function of ξ is replaced by a polynomial function to construct a generalized yield function for describing both asymmetry and Lankford coefficients under associated flow rule(AFR) framework, in which the linear combination of a series of ξ with odd powers is utilized to model the effect of materials' strength differential (SD). Since there are more parameters in the polynomial function of ξ than the function of ξ in those existing yield functions, and the independent basis functions ξ, ξ^2, ξ^3 … are informative enough to describe the SD effect and Lankford coefficients, the proposed yield function shows potential flexibility. The conditions for the convexity of the proposed yield criterion are obtained by using the method of order principal minor determinant. To describe sheet metals' anisotropic characteristics in tension and compression, the proposed isotropic criterion is further generalized to orthotropy through introducing the fourth-order linear transformation to the deviatoric stress tensor. To improve the flexibility of the yield criterion through introducing more fourth-order linear transformations, several extended proposed isotropic criteria are added together. The effectiveness and flexibility of the constructed yield criterion have been verified by applying to AA2008-T4(a BCC material) and AA2090-T3(a FCC material). Comparisons with Yld2000-2d and Yoon 2014 criteria show that the proposed yield criterion has the same ability as Yld2000-2d to describe metals' tensile properties and the proposed yield criterion can capture the SD effect like Yoon 2014, which validates that this generalized yield criterion can accurately describe not only the SD effect of metals but also the Lankford coefficients under uniaxial tensile loading and balanced biaxial tension based upon the AFR assumption. The flexibility of the proposed yield criterion is further validated by applying to zirconium through describing the evolution of yield surface for a clock-rolled plate with various levels of pre-strains during through-thickness compression. It is found that the proposed yield criterion can capture the asymmetry and the local deformation characteristic of the yield surfaces if the pre-strain is very large.

---

* Corresponding author.
** Corresponding author.

E-mail addresses: liheng@nwpu.edu.cn (H. Li), jun_chen@sjtu.edu.cn (J. Chen).

https://doi.org/10.1016/j.ijplas.2017.09.010
0749-6419/© 2017 Elsevier Ltd. All rights reserved.

Please cite this article in press as: Hu, Q., et al., A normalized stress invariant-based yield criterion: Modeling and validation, International Journal of Plasticity (2017), https://doi.org/10.1016/j.ijplas.2017.09.010

1. Introduction

Sheet metals usually demonstrate obvious anisotropy due to the complex mechanisms, for instance, Non-Schmid effect and twinning behaviors(Tuninetti et al., 2015; Patra et al., 2014). To accurately predict the strain and stress distribution is crucial for the researchers to investigate the formability and develop defect-free forming processes. Kinds of macroscopic yield functions(Hill, 1948; Hosford, 1972; Barlat et al., 1997, 2003, 2005; Barlat and Lian, 1989; Hu, 2005, 2007; Karafillis and Boyce, 1993; Banabic et al., 2005; Soare and Barlat, 2010; Yoshida et al., 2013; Aretz and Barlat, 2013) were proposed to describe the anisotropic behaviors for body centered cubic (BCC) and face centered cubic (FCC) structured metals. In order to increase the flexibility of orthotropic yield functions, two main approaches (Barlat et al., 2003, 2005; Yoshida et al., 2013; Yoon et al., 2014; Cazacu and Barlat, 2001, 2003; Cazacu et al., 2006) were adopted to introduce more anisotropic coefficients. One approach is based upon the theory of representations of anisotropic functions (Cazacu and Barlat, 2001), whereas another is the linear Cauchy stress tensor transformations (Sobotka, 1969; Boehler and Sawczuk, 1970) which is more popular since it's easy to promise the convexity condition of the yield function. Although many anisotropic yield functions in the past several decades have been developed under the hypothesis of associated flow rule (AFR), most of these functions are only suitable for either yielding prediction or Lankford coefficients prediction for some materials, which is particularly obvious for traditional anisotropic materials such as AA2090-T3(Park and Chung, 2012). To solve this problem, the non-associated flow rule (non-AFR) models, which describe the plastic flow and yielding direction with two independent functions were proposed (Stoughton and Yoon, 2004, 2006; Stoughton, 2002; Mohr et al., 2010; Cvitanić et al., 2008). However, it is still a top preference to develop a general yield function for practical usages under AFR framework for describing both the anisotropic yielding and Lankford coefficients.

All of the yield functions aforementioned are almost utilized to predict the anisotropic characteristic of FCC and BCC structured materials. These macroscopic yield functions are not suitable to describe the Hexagonal closed packed (HCP) structured metals since HCP materials show strong asymmetry for tension yield stress and compression yield stress with low strains. Even the most non-AFR models cannot describe the effect of strength differential (SD) since the yield functions in non-AFR models are symmetric. The noticeable SD effect are caused by the directionality of twinning for HCP structured materials, which has been frequently observed (Cazacu and Barlat, 2004; Choi et al., 2009; Khan et al., 2009; Stoughton and Yoon, 2004; Shutov and Ihlemann, 2012). The polycrystal viscoplasticity provide an effective way to predict the SD effect since it can be utilized to simulate the texture evolvement of HCP materials under various deformations (Proust et al., 2009; Jain and Agnew, 2007; Agnew et al., 2001). However, the applications of these approaches are unfortunately limited by the larger computation cost. Thus, to develop the new macroscopic plasticity models are greatly needed for the accurate and efficient simulations of complicated processes of sheet metal forming.

Considering the Drucker's criterion (Drucker, 1949) is an even function, it was modified as an odd function to take the SD effect of HCP metals into account by Cazacu and Barlat (2004). For pressure insensitive metals, another asymmetric yield function was developed by Cazacu et al. (2006) through applying a linear transformation, which is called as the CPB06 yield function. The effectiveness of CPB06 yield function was validated by Yoon et al. (2013) and Plunkett et al. (2007) through describing the anisotropic hardening of high-purity zirconium and AZ31. To describe the evolution of the mechanical response of AZ31B-O, the extended CPB′06 yield function was developed by Tari et al. (2014). Nixon et al. (2010) and Plunkett et al. (2007) used the interpolation between adjacent yield surface at different levels of accumulated strain to describe the anisotropic yield surface at arbitrary given equivalent plastic strain. This anisotropic yield function was applied to α-titanium to capture the SD effects which is associated to twinning. The interpolation approach was also adopted by Li et al. (2016) through introducing CPB′06 yield function and Yoon's criterion (Yoon et al., 2014) to describe the evolution of distortional yield surface of high strength titanium alloy. Considering the effects of temperature and strain rate, Khan and Yu (2012) experimentally investigated the thermal mechanical characteristics of Ti–6Al–4V alloy in both compression and tension modes. An asymmetric yield function related to temperature and strain rate was developed by Khan et al. (2012) to describe the SD effects of Ti–6Al–4V alloy by introducing the Khan–Huang–Liang constitutive model. Yoon et al. (2014) proposed a new yield criterion for pressure sensitive metals by introducing the first stress invariant based upon the asymmetric yield function proposed by Cazacu and Barlat (2004). However, the convexity of Yoon's criterion cannot be promised since the two linear transformations are assigned on two subtract terms.

These yield functions to describe the SD effect discussed above are usually used with non-associated flow plasticity. Besides, under non-AFR, the complicated constitutive models are difficult to implement since the equivalent plastic strain relation (Safaei et al., 2015). Under the assumption of AFR, the description about yielding direction is not accurate, since the parameters in the yield functions are determined just by the yield stresses without the Lankford coefficients, which leads to incorrect prediction about the Lankford coefficients during plastic deformation. If the yield function can not only accurately describe the SD effect of metals but also the Lankford coefficients based on AFR, the evolution of yield stress and strain during plastic deformation can be reasonable predicted by choosing the proper constitutive model.

As discussed above, although the flexibility of some yield functions has been increased, these orthotropic yield functions cannot be adopted under AFR assumption to describe both the Lankford coefficients and the characteristic of tension-compression asymmetry, which necessitates the development a generalized yield function with more accuracy and flexibility for predicting the anisotropy and asymmetry of FCC, BCC and HCP structured metals under AFR assumption. In our present work, a generalized yield criterion within the AFR framework is developed and further extended to orthotropy by introducing linear transformation tensors. The influences of the parameters in the proposed yield criterion on prediction

Please cite this article in press as: Hu, Q., et al., A normalized stress invariant-based yield criterion: Modeling and validation, International Journal of Plasticity (2017), https://doi.org/10.1016/j.ijplas.2017.09.010