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Material length scale of strain gradient plasticity: A physical interpretation

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ABSTRACT

The physical basis and the magnitude of the material length scale in theories of strain gradient plasticity are crucial for accounting for size effects in the plastic behavior of metals at small scales. However, the underlying physics of the length scale is ambiguous. The length scales in strain gradient plasticity theories in which the plastic work density can be expressed as a function of the gradient-enhanced plastic strain are here derived from known physical quantities via critical thickness theory. A connection between the length scale and the fundamental physical quantities is elucidated. The combination of the strain and strain-gradient terms within the deformation theory of strain gradient plasticity is addressed. It is shown that, compared with the harmonic sum of the strain and strain-gradient terms in Fleck-Hutchinson theory, the linear combination gives a more reasonable value of length scale, several micrometers, which is close to that in the gradient theory of Aifantis. In contrast, the value of length scale in Nix-Gao theory is much larger, in the millimeter range. The length scales determined by critical thickness theory are in good agreement with those obtained by fitting to experimental data of wire torsion.

1. Introduction

A wide range of experiments at the micron/sub-micron scale have revealed strong size-dependent strengthening associated with plastic strain gradients induced by non-uniform deformation, for example, nano indentation (e.g. Ma et al., 2012; Stelmashenko et al., 1993; Swadener et al., 2002; Zhu et al., 2008), thin wires in torsion (e.g. Fleck et al., 1994; Dunstan et al., 2009; Liu et al., 2013a, b; Liu et al., 2012), thin foils in bending (e.g. Ehrler et al., 2008; Haque and Saif, 2003; Hayashi et al., 2011; Moreau et al., 2005; Stölken and Evans, 1998), etc. In general, the observed phenomenon is that smaller is stronger, which is referred to as the size effect. These experimental observations cannot be captured by conventional theories of plasticity since such theories do not involve any characteristic length scales. In parallel, inspired by the concept of geometrically necessary dislocations (GNDs) introduced by Nye (1953), Kröner (1958), Lardner (1969) and Ashby (1970), various continuum theories of small-scale plasticity, e.g. strain gradient plasticity (SGP) theory (e.g. Abu Al-Rub and Voyiadjis, 2006; Aifantis, 1987, 1999; Brinckmann et al., 2006; Ban et al., 2017 Fleck and Hutchinson, 1997, 2001; Fleck et al., 1994; Fleck and Willis, 2009a, b; Gao et al., 1999; Gudmundson, 2004; Gurtin and Anand, 2005a, b; Hutchinson,

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2012; Kuroda and Tvergaard, 2010; Mühlhaus and Aifantis, 1991; Nix and Gao, 1998; Voyiadjis and Faghihi, 2012; Voyiadjis et al., 2010; Lubarda, 2016; Zreid and Kaliske, 2016) have been proposed for encapsulating the observed size effects in non-uniform deformation. These theories are often based on the gradient-enhanced effective plastic strain variable (Evans and Hutchinson, 2009; Hutchinson, 2012; Lubarda, 2016; Niordson and Hutchinson, 2011). However, most of the aforementioned models neglect the contribution of the plastic spin (i.e. the skew-symmetric part of the plastic distortion), which may be problematic in certain situations (Bardella, 2010; Bardella and Panteghini, 2015; Poh et al. 2011). The gradient plasticity theory proposed by Gurtin (2004) is different from most of the other SGP theories available. It accounts for the dissipation arising from the plastic spin together with the energetic counterpart in the free energy, i.e. the *defect energy*. The recent literature (e.g. Bardella, 2009, 2010; Bardella and Panteghini, 2015; Poh and Peerlings, 2016; Wulfinghoff, 2017) has demonstrated its relevance in isotropic gradient plasticity theory, even on the basis of the comparison with strain gradient crystal plasticity of Bardella (2006). In particular, by analyzing the size effect by the deformation approximation of Gurtin's theory, Bardella (2010) found that the size effect due to the defect energy leads to strain hardening [1] with diminishing size, while the gradient-enhanced plastic potential gives rise to strengthening, i.e. the increase in initial yielding. Besides, various theories of strain gradient crystal plasticity (e.g. Bammann, 2001; Ma et al., 2006; Clayton et al., 2006; Gurtin and Needleman, 2005; Gurtin, 2008; Gurtin and Reddy, 2014) and continuum dislocation theory (Berdichevsky, 2006, 2016; Le, 2016; Le and Günther, 2014; Zaiser, 2015) have also been developed for modeling the size-dependent plastic behavior.

To date, strain gradient plasticity has attracted much attention due to its feasibility in applications. Most of SGP theories are phenomenological isotropic theories (e.g. Aifantis, 1987, 1999; Fleck and Hutchinson, 1993, 1997, 2001; Gudmundson, 2004; Gurtin, 2004; Gurtin and Anand, 2005a, b; Hutchinson, 2012; Mühlhaus and Aifantis, 1991); Several of them are based on physical dislocation mechanisms, for example, the mechanism-based strain gradient plasticity (MSG) theory (Gao et al., 1999; Nix and Gao, 1998), the physically-based gradient plasticity theory (Abu Al-Rub and Voyiadjis, 2006), and the dislocation-density based strain gradient model (Brinckmann et al., 2006). An early attempt for strain gradient plasticity is to extend the rate-independent J_2 theory by involving a dependence on plastic strain gradients, e.g. Mühlhaus and Aifantis (1991), Fleck and Hutchinson (1993, 2001). However, the authors do not discuss the compatibility of the theories with thermodynamic requirements in detail. Gudmundson (2004) and Gurtin and Anand (2009) pointed out that the Fleck-Hutchinson theory (2001) does not always meet the thermodynamic dissipation restriction. Afterwards, Hutchinson (2012) and Fleck *et al.* (2014, 2015) modified the theory to meet the thermodynamic requirement through partitioning the higher-order stresses into energetic (recoverable) and dissipative components. Alternatively, another attractive class of phenomenological SGP theories which are thermodynamically consistent have been established by Gudmundson (2004), Gurtin (2004) and Gurtin and Anand (2005a). These theories express the higher-order stresses in terms of the increments of strain and strain gradient. Recently, Hutchinson (2012), Fleck et al. (2014, 2015), and Bardella and Panteghini (2015) indicated that this expression can lead to the possibility of elastic gap in the plastic flow, which requires experimental validation (Fleck et al., 2015).

In this paper, we merely focus on the class of SGP theories in which the plastic work density can be expressed as a function of a gradient-enhanced plastic strain measure, see the discussion of Evans and Hutchinson (2009), Fleck et al. (2014, 2015), Hutchinson (2012), Lubarda (2016), and Niordson and Hutchinson (2011). In these theories, the constitutive relation allows for a contribution from the local value of plastic strain and a contribution from the local plastic strain gradient. For dimensional reason, one or more length scales necessarily enter into the theories. One of reasons why SGP theories are able to successfully fit experimental data at small scales is that the length scales introduced are free fitting parameters. However, the physical basis of the material length scale [2] in SGP is ambiguous (Chakravarthy and Curtin, 2011; Dunstan, 2016; Evans and Hutchinson, 2009; Groma *et al*., 2007); that is one of the reasons why the SGP theories have not been thoroughly accepted (Evans and Hutchinson, 2009). To date, only a few attempts (e.g. Abu Al-Rub and Voyiadjis, 2006; Dunstan, 2016; Evans and Hutchinson, 2009; Groma et al., 2007; Voyiadjis and Abu Al-Rub, 2005; Zhang and Aifantis, 2015) have been made to study its physical origin. For example, by fitting to the bending data of thin foils, Evans and Hutchinson (2009) found that the length scale in the Nix-Gao (NG) model is about 25 mm, and in the Fleck-Hutchinson (FH) model it is about 5 μm, but the physical interpretation of these values is not clear. Here we will provide an interpretation of these very divergent values.

As well as the ambiguities about the physical interpretation of the material length scales in SGP, another reason for doubt is the experimental evidence for a size effect at yield. The interpretations of SGP are always in terms of GNDs, yet at initial yield there is not yet any plastic strain gradient and hence no (or only a few) GNDs. Evans and Hutchinson (2009), however, showed that strain gradient plasticity can predict a size effect in the form of an elevated yield stress. By considering the predictions for perfect plasticity, they showed that the FH theory rather surprisingly predicts an elevated yield stress and no size-effect-induced strain hardening, while the NG theory does the opposite as expected. Evans and Hutchinson (2009) were not able to explain the strengthening convincingly in physical terms.

In semiconductor technology, critical thickness theory (CTT) has been well developed in the context of epitaxial strained layers (e.g. Matthews, 1966; Fitzgerald, 1991; Dunstan, 1997) and other films (Nix, 1989), and CTT gives a physical insight into

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[1] That is an increase of the flow stress required to attain a given amount of plastic strain.

[2] It was called the “ghost length scale” by Dabiao Liu and Xu Zhang in the Sino-Hellenic workshop organized by Prof. E. C. Aifantis in 2015.