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A numerical approach to nonlinear propagation of light in photorefractive media

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ABSTRACT

The article presents a new approach to the analysis of light propagation in photorefractive materials. The discussed numerical method can be used for an analysis of the dynamics of nonlinear effects taking place in those media in which an analytical approach requires the use of approximations or is impossible. As an example of how the method works, the results of simulation are shown, illustrating the process of spatial solitary wave formation in two materials: a photorefractive semiconductor and a ferroelectric crystal.

1. Introduction

Photorefractive materials have been drawing much interest since first research publications describing them appeared [1]. This interest is connected with highly promising prospects for their applications, particularly in the field of all-optical processing and switching of light [2–4]. One of the main fields of promising research now underway is the generation of photorefractive solitary waves [5,6] and an analysis of possibilities to control them by the management of their trajectories [7–9] or soliton interactions [10,11]. Notably, such research is often carried out in the context of specific applications—for example, in optical telecommunications [12–14].

Interesting potential applications direct research towards seeking new materials, which broadens the family of media displaying the photorefractive effect. So far, the effects connected with self-trapping of light have been investigated or observed in such materials as: ferroelectric oxides [15], sillenites [16], paraelectrics [17], photorefractive polymers [18], organic glasses [19] and semiconductors [20,21]. The diversity of photorefractive media is due to complexity and dissimilarity of physical processes taking place in particular materials. This, in turn, necessitates expanding theoretical models of the photorefractive effect in various materials. The simplest strategy within the band transport model frames accounts for one type of carriers (most often electrons) excited from a single deep-level dope (most often donors). However, in the case of many interesting photorefractive materials this strategy proves to be insufficient. It is often necessary to account for the transport of both electrons and holes [22], additional doping levels [23,24] or in the case of semiconducting materials, the hot electron phenomenon [25–27].

In terms of mathematics, the relations describing the photorefractive process in a given material usually form a system of nonlinear partial differential equations, strongly coupled with each other. In the case of both typical materials (for which the photorefractive effect can have the simplest model), and those that are more complex, an analytical solution of this equation system is possible only through approximations, or is just impossible. An alternative way is the use of numerical methods which enable accurate solutions that, fill in the blanks arising in the analytical approach, and providing a verification of the approximations used so far.

This article shows how the development and combination of known numerical algorithms may be used for an effective numerical analysis of nonlinear light propagation in photorefractive materials, both typical ones and those requiring more complex models. The presented results have been obtained using an algorithm that in the author's opinion may be relatively easily modified as well as developed in its implementation phase. A reference point is semiconducting materials whose relatively complex model is based on transport of both electrons and holes, and which accounts for the hot electron phenomenon. For a comparison, the results of calculations made for a standard model describing the propagation of solitons, for example in selected ferroelectric oxides are presented as well.

2. Theoretical background

One characteristic feature of the photorefractive effect is that it may take place in media where free charge carriers are excited by

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light. Excited carriers move by diffusion and under the influence of external electric field, until the moment of their recombination with traps, which is most likely to take place in unlighted areas. If the light falling on the material has non-uniform intensity, it will cause a non-uniform spatial charge distribution, resulting in the appearance of internal electric field. This field causes the refractive index to change through the electro-optic effect, which influences the propagation of optical beam.

The simplest mathematical approach to the photorefractive effect is based on the so-called Kukhtarev–Vinetski model [28]. In its original form this model reflects a weakly doped dielectric medium, where the charge carriers are electrons, optically excited from a deep donor level. In such case an inequality $N_D > N_A$ is fulfilled, where N_D is donor concentration and N_A is shallow acceptor concentration. The equation system describing the photogeneration, transport and trapping of carriers takes the following form:

$$\frac{\partial N_D^+}{\partial t} = (\beta_n + S_n I)\left(N_D - N_D^+\right) - \gamma_n n N_D^+, \tag{1a}$$

$$\mathbf{J_n} = q\mu_n n\mathbf{E} + \mu_n k_B T \mathrm{grad} n, \tag{1b}$$

$$\frac{\partial}{\partial t}\left(N_D^+ - N_A - n\right) = \frac{1}{q}\mathrm{div}\mathbf{J_n}, \tag{1c}$$

$$\mathrm{div}\mathbf{E} = \frac{q}{\varepsilon\varepsilon_0}\left(N_D^+ - N_A - n\right), \tag{1d}$$

where n describes electron concentration, β is a coefficient of thermal generation of carriers from deep traps, γ is a coefficient of recombination of carriers with traps, $S = \delta/h\nu$ is photoionization cross-section divided by photon energy, μ—carrier mobility, J—current density, E—electric field intensity inside a medium, I—light intensity distribution, q—elementary charge, k_B—Boltzmann constant, ε_0 is the vacuum permittivity, ε is the dielectric constant, T—temperature, N_D^+—ionized donor concentration.

In order to make an analysis of nonlinear light propagation in a material of such characteristics, the equation system (1a)–(1d) has to be supplemented with a nonlinear wave equation. For a two-dimensional case $(1+1)D$, the electrical component of the electromagnetic wave propagating in the direction of the z-axis:

$$\mathbf{E}_{\mathrm{opt}}(x, y, z, t) = \mathbf{A}(x, y, z)\exp(i\omega t - ikz),$$

fulfills the nonlinear propagation equation:

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k}\nabla_\perp^2\right)\mathbf{A} + \frac{ik}{n_0}\mathbf{\Delta n A} = 0, \tag{2}$$

where $k = 2\pi n_b/\lambda$ is wave number, $\omega = 2\pi c/\lambda$ is angular frequency, Δn is the optically induced change of refractive index n_b. Approximate steady-state solutions of equation system (1) together with an analysis of photorefractive soliton states have been presented in the works [5,6], while time-dependent solutions are outlined in [29,30]. The analysis of both steady state and temporal states is based on approximations justified in the case of many typical materials, which allows us to reduce the problem described by Eqs. (1)–(2) to a single nonlinear differential equation. Further research into nonlinear light propagation is most often carried out by numerical analysis of the nonlinear propagation equation [31,32] obtained for the examined problem. Unfortunately, for many interesting optical materials the photorefractive effect has to be described using more complex models than the one outlined by equation system (1). In those cases the reduction of the nonlinear propagation problem to a single differential equation may be difficult, or will require significant approximations limiting a full analysis. In such cases, the problem can be solved using a fully numerical approach. Interesting and different than presented in this paper numerical models, have been shown in the works [33,34].

Photorefractive semiconductors are an interesting example of materials where the photorefractive process is more complex than the one described by equation system (1). Basic differences result from a more complex transport of carriers, which has a bipolar character in this particular case. The presence of both electrons and holes has been discussed mostly in the works devoted to analyses of photorefractive gratings [22–24,27] and to a lesser but still significant extent, to the phenomena involved in nonlinear light propagation [35–37]. One additional complication appearing in the analysis of photorefractive transport in semiconductors is the "hot-electron" effect. Semiconductors such as GaAs or InP feature a conduction band having a distinct central minimum Γ with high mobility of carriers, and higher located (but still close) side valleys L characterized by lower mobility. When an electric field having intensities exceeding some critical value E_C is applied to such media, the result is that some of the electrons heated by the field are transferred from valley Γ to valley L. As a result, the relation between electron velocity and the electric field intensity becomes strongly nonlinear in the range of electric fields whose intensities exceed the critical value. Therefore, the equation system describing the photorefractive process in such materials takes into account both the electron and hole transport and the dependence of electron mobility and temperature on electric field intensity [27,36,37].

If we assume that acceptors are deep traps, for semiconductors the equation system describing the photogeneration, transport and trapping of carriers may take the following form:

$$\frac{\partial n}{\partial t} - \frac{1}{q}\mathrm{div}\mathbf{J_n} = bI + (S_n I + \beta_n) N_A^- - \gamma_n n\left(N_A - N_A^-\right), \tag{3a}$$

$$\frac{\partial p}{\partial t} + \frac{1}{q}\mathrm{div}\mathbf{J_p} = bI + \left(S_p I + \beta_p\right)\left(N_A - N_A^-\right) - \gamma_p p N_A^-, \tag{3b}$$

$$\mathbf{J_n} = q\mu_n(\mathbf{E})n\mathbf{E} + k_B \mathrm{grad}\left[\mu_n(\mathbf{E})T_n(\mathbf{E})n\right], \tag{3c}$$

$$\mathbf{J_p} = q\mu_p p\mathbf{E} - k_B\mu_p T_L \mathrm{grad} p, \tag{3d}$$

$$\frac{\partial}{\partial t}\left(n + N_A^- - p - N_D^+\right) = \frac{1}{q}\mathrm{div}\left(\mathbf{J_n} + \mathbf{J_p}\right), \tag{3e}$$

$$\mathrm{div}\mathbf{E} = -\mathrm{div}\left(\mathrm{grad}\varphi\right) = \frac{q}{\varepsilon\varepsilon_0}\left(N_D^+ + p - n - N_A^-\right), \tag{3f}$$

where n describes the concentration of electrons, p describes the concentration of holes, $\mu_n(E)$ is the mobility of electrons depending on the electric field, $T_n(E)$ is the temperature of electrons, μ_p is the mobility of holes, T_L is a lattice temperature, while φ is the electrical potential in the material, and N_A^- the ionized acceptor concentration. Parameters S and β are defined similarly to the case in Eqs. (1). The model additionally assumes that the carriers can be excited not only in transitions from the trap level, but also in inter-band transitions [33,34]. The value of b describes the absorption coefficient corresponding to inter-band transitions.

The electrons heated by the electric field reach a temperature which may significantly exceed the lattice temperature T_L. Its value may be formally determined using this relation [3]:

$$T_n(E) = T_L + \frac{2q\tau_r v_n(E)}{3k_B}E, \tag{4}$$

where $v(E)$ is an electron drift velocity depending on the electric field, τ_r is an average energy relaxation time with values ranging from 0.1 to 1 ps [25,26]. The relation between electron mobility and electric field intensity in the twin valley model may be expressed as a weighted average:

$$\mu_n(E) = \mu_{nl} f(E) + \mu_{nu}[1 - f(E)], \tag{5}$$

where μ_{nl} and μ_{nu} are, respectively, carrier mobilities in central and side valleys, while $f(E)$ is a distribution function describing the population of the central valley with the following relation:

$$f(E) = \left\{1 + R\exp\left[\frac{-\Delta U}{k_B T_n(E)}\right]\right\}^{-1} \tag{6}$$

where ΔU is an energy difference between the central and side minimum, while R is a ratio of state density in the central and side valleys. For gallium arsenide $R = 94$, $\Delta U = 0.31$ eV.