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Shakedown of porous materials

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ABSTRACT

The paper is devoted to the determination of shakedown limit states of porous ductile materials under cyclically repeated loads by considering the hollow sphere model with von Mises matrix. We adopt Melan's statical approach based on time-independent residual stress fields. First of all, we determine the exact solution for the pure hydrostatic loading. It turns out that the collapse occurs by fatigue. Next, suitable trial stress fields are built with additional terms to capture the shear effects. This theoretical fatigue criterion of porous materials agreed with numerical simulations and experimental results provided in literature according to which, in ductile metals, the strain to fracture under cyclic loading is considerably lower than the one reached monotonically. Finally, the built criterion is compared to numerical simulations derived by both shakedown direct method and incremental simulations.

1. Introduction

To determine limit loads for materials and structures operating beyond the elastic limit is since ever one of the most important tasks of engineers. For this purpose, the so-called Direct Methods play an increasing role due to the fact that they allow rapid access to the requested information in mathematically constructive manners. They embrace limit analysis, the oldest and most developed approach, and shakedown analysis, a powerful extension to the variable repeated loads potentially more economical than step-by-step inelastic analysis (Maier et al., 2000; Save et al., 1997).

The limit analysis allows to predict the ultimate load α^L, called the limit load. This theory, developed intuitively in the 1930s, theoretically and experimentally based in the 1950s, is now widely used and has become recommended by design Codes on pressure vessels (ASME) and on reinforced concrete slabs (European Committee for Concrete).

However, in the case of variable repeated loads, the magnitude of the ultimate load is not the only factor characterizing the structural safety, as depicted in Fig. 1. Let α be a positive real number governing the size of the load domain, said the load factor. The experimental tests show that, in some cases, above a particular value α^{SD} of the load factor, we can observe after a transient phase an accumulation of plastic strains leading to excessive deformations. This kind of failure is called incremental collapse, ratchet or ratchetting. Collapse by development of a mechanism, as in limit analysis, can be considered as the particular case of ratchet where the collapse occurs during the first cycle.

On the contrary, below the value α^{SD}, we observe after the transient phase a stabilization of the plastic deformations. For such an event, the response of the structure to the cyclic loading becomes purely elastic. This behavior is called (elastic) shakedown or adaptation. We say that the structure shakes down.

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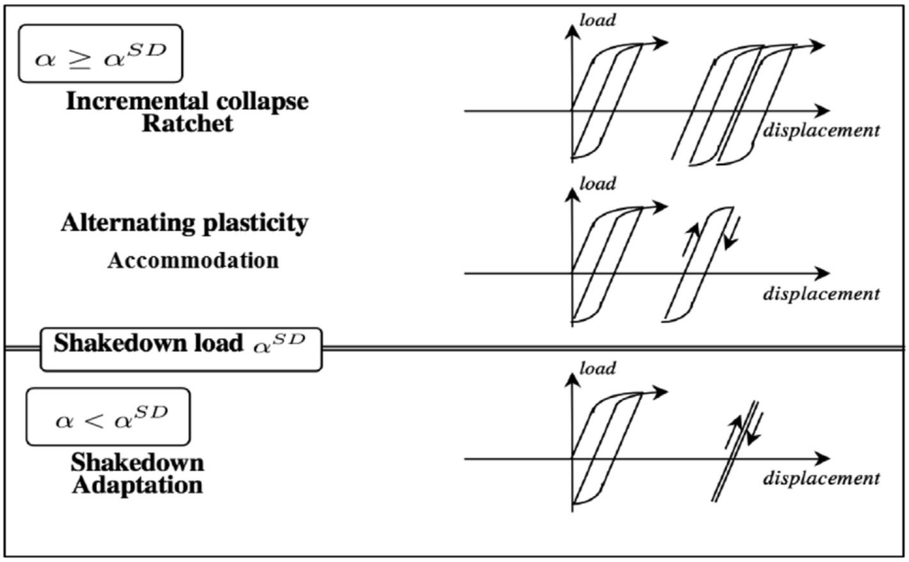

Fig. 1. Incremental collapse, alternating plasticity and shakedown.

For other structures subjected to variable loading, the deformation remains small but it can be observed that, beyond a characteristic value α^{SD}, alternating plastic strain increments occur after the transient phase, leading to fracture after a number of cycles (Coffin, 1954; Manson, 1954). The behavior observed above the load α^{SD} is called alternating plasticity, accommodation or plastic shakedown.

Therefore the structural safety requires that the power dissipated by the plastic deformation due to repeated load or temperature changes should eventually cease. The corresponding safety load α^{SD} is called shakedown load. Above this value, the structure fails by ratchet or alternating plasticity. Below α^{SD}, the structure shakes down, and the total dissipation is bounded in time.

It is worth to remark that the duration of the transient phase increases with the degree of redundancy of the structure and is highly variable, from one cycle to a few hundred or even a few thousand cycles. A complete step-by-step computation of the overall history up to the limit state (see for instance (Abdel-Karim, 2009, 2010; Taleb and Hauet, 2009)) can be very time-consuming or even exceed the capacity of the computer. On the contrary, the shakedown analysis allows a direct access (see (Chinh, 2007; Polizzotto, 2010; Simon, 2013)) to the gist of the information useful for engineers, the value of the shakedown factor below which the structural safety is guaranteed. Besides, if the loading path is unknown, then shakedown analysis *must* be used.

As in limit analysis, there are two dual theoretical approaches. The statical one, with the key-concept of time-independent residual stress field, was introduced historically the first. Papers by Bleich (1932), Melan (1936) and Symonds (1951) are considered as the starting point of the method. The kinematical approach and the basic concepts of admissible plastic strain increment were introduced by Koiter (1960) and developed by Neal (1956), Gokhfeld (1956) and Sawczuk (1969). An improved formulation including the concept of loading factor was proposed by Martin (1975). For more details and historical view on the shakedown theory, the reader is referred to (Weichert and Ponter, 2014).

The idea to apply shakedown concepts to the fatigue of materials is due to Dang Van (1973). Starting with the pioneering Orowan's paper (1939) on grain plasticity, he states that fatigue does not occur if all grains reach an elastic shakedown state. In order to estimate the stress-strain state at the meso scale, a simple homogenisation scheme of a plastic inclusion in an elastic matrix is considered by Dang Van (1993), Papadopoulos (1994), Charkaluk et al. (2009).

With the development of computers, numerous numerical approaches were proposed for the homogeneization by direct methods, mainly for periodic materials. Concerning the limit analysis, one can quote the computation of the overall yield stress for soils reinforced by periodic linear inclusions (de Buhan and Hassen, 2013) and periodically heterogeneous plates (Bleyer et al., 2015). Concerning the shakedown analysis, Hachemi et al. (2014, 2000) proposed a numerical methodology to investigate the failure of periodic composite materials under thermo-mechanical variable loads in presence of plasticity and damage. In Chen et al. (2013), it is combined with interior point method for efficient computation. In Chen et al. (2015), extended this methodology to random material to compute the endurance limit of a particle reinforced metal matrix composite (PRMMC). In Magoariec et al. (2004), Magoariec et al. apply the shakedown analysis to 3D periodic heterogeneous media. In Liu et al. (2005), the static shakedown theorem is reformulated making use of the symmetric Galerkin boundary element method (SGBEM).

In the framework of the micromechanics of ductile porous media, Gurson proposed in a famous paper (Gurson, 1977) a closed analytical formula for the upper bound limit analysis approach of a hollow sphere having a von Mises solid matrix, with

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