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A finite strain viscoplastic constitutive model for rubber with reinforcing fillers

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ABSTRACT

A three dimensional viscoplastic constitutive model for finite strains in a co-rotational explicit scheme is developed and implemented using finite elements that captures the amplitude dependency, commonly referred to as the Fletcher-Gent effect, and frequency dependency of rubber with reinforcing fillers. The multiplicative split of the deformation gradient is utilized and the plastic flow rule stems from an extension to finite strains of a boundary surface model with a vanishing elastic region. The storage and loss modulus for a 50 phr carbon black filled natural rubber are captured over a large range of strain amplitudes, 0.2–50% shear strain, and frequencies, 0.2–20 Hz. In addition, bimodal excitation is replicated accurately, even though this measurement data is not included when obtaining material parameters. This capability is essential when non-sinusoidal loading conditions are to be replicated. By separating the material and geometrical influence on the properties of a component, the design engineers have the capability to evaluate more concepts early in the design phase. This also reduces the need of complex prototypes for physical testing, thereby saving both time and money.

1. Introduction

Vulcanized rubber with reinforcing fillers is a fascinating material with unique material properties such as a high elongation at break and a remarkable fatigue resistance (Heinrich et al., 2002; Höfer and Lion, 2009). It is used in several industrial applications, including the automotive and aerospace industry, as shock absorbers, bushings and vibration isolators (Rendek and Lion, 2010). Early in the design process, engineers need reliable tools and methods to assess the reliability and properties of these components. Early simulations has the potential to replace some physical testing, thereby reducing the cost and development time of new components. However, the results from such calculations are limited by the accuracy of the material models employed.

Pure rubber vulcanizates, well above the glass transition temperature of the material, are accurately captured by statistical and continuum mechanics under quasi-static loading conditions (Boyce and Arruda, 2000; Ogden, 1973; Holzapfel, 2006; Edwards and Vilgis, 1998; Marckmann and Verron, 2006). Interestingly enough, the addition of reinforcing fillers, the most common being carbon black (Heinrich and Klüppel, 2002), introduces a strain amplitude dependency on the storage and loss

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modulus of the material, for strain amplitudes larger than 0.01–0.1% (Fletcher and Gent, 1953; Heinrich and Klüppel, 2002; Payne, 1962). This is the well known Fletcher-Gent effect, and the storage modulus at small strain amplitudes compared to moderate strain amplitudes can be an order of magnitude higher (Rendek and Lion, 2010). Moreover, rubber with reinforcing fillers exhibits hysteresis even at very low strain rates. Most importantly, these materials are not linear in the sense that the sum of the response from two individual sinusoidal excitations does not equal the response from when the sinusoidal excitations are performed simultaneously (Wrana and Härtel, 2008; Sjöberg and Kari, 2003). Therefore, the superposition principle is not valid and constitutive models in the frequency domain are inherently complicated, and a material model for this class of materials is advantageously formulated in the time domain.

In addition, rubber is weakly strain rate dependent, and a single relaxation time is insufficient to characterize the material over a wider frequency range (Adolfsson et al., 2005; Rendek and Lion, 2010; Höfer and Lion, 2009). Two modelling techniques to capture this behaviour are commonly found in the literature. Firstly, several material parameters are used in order to recreate the spectrum of relaxation times found from experiments (Govindjee and Reese, 1997; Reese and Govindjee, 1998; Austrell and Olsson, 2001; Olsson and Austrell, 2003), and secondly, fractional derivatives are employed which significantly reduces the number of parameters (Bagley and Torvik, 1983; Adolfsson and Enelund, 2003; Adolfsson et al., 2004; Kari, 2003; García Tárrago et al., 2007). Fractional derivatives have a major drawback in the time domain in that the state variables from all previous time steps are needed in order to calculate the next increment. This results in an exponential increase in computational cost for longer simulations in the time domain, unless a sparse time history is incorporated (Adolfsson et al., 2004). Regardless, neither modelling approach is, without modification, capable of capturing the Fletcher-Gent effect. One solution for recreating the amplitude dependency is to incorporate internal variables and non-linear viscoelastic evolution laws, either with an intrinsic time scale (Rendek and Lion, 2010; Höfer and Lion, 2009) or where the viscous flow rule is a function of both the current strain and stress (Bergström and Boyce, 1998; Ayoub et al., 2014). Nevertheless, it has proven difficult to capture both the storage and loss modulus over a wide range of strain amplitudes and frequencies with the same set of material parameters (Rendek and Lion, 2010; Höfer and Lion, 2009; Bergström and Boyce, 1998).

Instead, alternative approaches to model the Fletcher-Gent effect exist. In phenomenological uniaxial implementations this could be in the sense of a smooth frictional element (Berg, 1998; Sjöberg and Kari, 2002; García Tárrago et al., 2007; Coveney and Johnson, 2000), whereas physically motivated models aim to capture the interactions between polymer and filler, and the breakdown of the filler network (Raghunath et al., 2016; Bergström and Boyce, 1999; Xi and Hentschke, 2012; Wulf and Ihlemann, 2013). For both cases, uniaxial material models can be extended to three dimensions for instance with the concept of representative directions and the integration over a micro-sphere (Dargazany et al., 2014; Miehe et al., 2004). A further possibility is to incorporate the mathematical framework of plasticity as the three dimensional equivalence of a frictional element (Netzker et al., 2010; Besdo and Ihlemann, 2003a), as has been done in this paper. In order to capture the strain-rate dependency, a viscoelastic contribution can be added in parallel to the plasticity element (Berg, 1998; Sjöberg and Kari, 2002; García Tárrago et al., 2007; Coveney and Johnson, 2000; Miehe and Keck, 2000; Austrell and Olsson, 2001; Olsson and Austrell, 2003; Martinez et al., 2011), and via a rate-dependency on the frictional elements (Coveney and Johnson, 2000; Hu and Wereley, 2012). However, experiments in literature show that the strain-rate dependency is larger for small strain amplitudes than for large amplitudes (Coveney and Johnson, 2000; Österlöf et al., 2014; Rendek and Lion, 2010; Höfer and Lion, 2009). This motivates placing the viscoelastic element in series with a plastic element, as was done in previous work by the authors (Österlöf et al., 2014). In this uniaxial implementation, both the storage and loss modulus is captured over several decades of strain amplitudes and frequencies, using only a few material parameters. More importantly, the response from a bimodal excitation is replicated, even when the bimodal experimental data is not used for obtaining the material parameters.

The aim of this paper is to derive a three dimensional material model for finite strains with the same modelling capabilities. With such a model, the influence from material and geometry on the properties of a component can be separated. This enables realistic simulations early in the design phase, with material parameters derived from simple material tests.

Viscoplasticity is a vast research area, and in order to proceed, some key features and objectives of the scope of this work are introduced. First and foremost, finite strains need to be taken into account, whereby the additively decomposition of the total strain ∈ into an elastic part and inelastic part, $\varepsilon = \varepsilon_{\text{el}} + \varepsilon_{\text{inel}}$, is invalid. Instead, the multiplicative decomposition of the deformation gradient, as presented by Lee (1969) will be utilized. This decomposition is moreover micro-mechanically motivated (Reese, 2003). Secondly, overlay models in which the total stress is a sum of viscoelastic and elastoplastic contributions, such as in Austrell and Olsson (2001); Olsson and Austrell (2003); Kim and Muliana (2009); Bardenhagen et al. (1997); Haupt and Sedlan (2001); Miehe and Keck (2000); Lion (2000) and Muhr (2010) are at disadvantage since the strain rate dependency is not additive, as discussed in the previous paragraph. In addition, it is assumed that the material exhibits a hysteresis at infinitely slow loading rates, due to the breakdown and reforming of the filler network (Kraus, 1984; Heinrich and Klüppel, 2002; Donnet and Custodero, 2013; Lion, 1996; Papon et al., 2012). Therefore, the mathematical framework of plasticity is deemed reasonable, as opposed to material models where no equilibrium hysteresis is assumed to exist (Bergström and Boyce, 1998). Furthermore, it is acknowledged that the underlying physical reason for hysteresis in rubber with reinforcing fillers is fundamentally different from the yield behaviour in metals, for which most of the research in viscoplasticity has been conducted, see for instance Scheidler and Wright (2001); Gurtin and Anand (2005a, b); Bodner and Partom (1975); Xiao et al. (2006); Horstemeyer and Bammann (2010) and references therein.

Finally, many rubber vulcanizates exhibit a strain softening, known as the Mullins effect, in which the stress depends on the maximum previously experienced strain. For rubber with reinforcing fillers this effect may be considerably pronounced

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