International Journal of Pressure Vessels and Piping 155 (2017) 1–14

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Analytic solution of effect of electric field on elasto-plastic response of a functionally graded piezoelectric hollow cylinder

Ali Asghar Atai [a], Davood Lak [b, *]

[a] School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran
[b] National Iranian Oil Co., Tehran, Iran

ARTICLE INFO

Article history:
Received 18 April 2016
Received in revised form
9 June 2017
Accepted 12 June 2017
Available online 15 June 2017

Keywords:
Elasto–plastic analysis
FGPM
Hollow cylinder
Pressure vessel
Electric response

ABSTRACT

An analytical technique is developed to obtain the effect of electric potential on mechanical and electrical response of a functionally graded piezoelectric (FGP) hollow cylinder under action of internal and external pressure and temperature gradient. It is assumed that properties of FGP material are changed through thickness according to power law function. The exact solution is done in two elastic and plastic sections and stresses, strains and displacements are obtained. It is shown in two examples that for a pressure vessel under interior pressure and thermal loads in electric potential field, yielding may commence from the outside, or from inside and outside simultaneously, depending on material in-homogeneity and combination of loads and the plastic area is extended towards inside with the increase of electric potential.

1. Introduction

Functionally graded piezoelectric material is a kind of piezoelectric material combined of two constituents which continuous transition between one constituent to another in a certain gradient. This property provides a great potential for use in cylindrical vessels in advanced applications like aerospace structures and transducers.

In recent years, functionally graded material attracted great attention from several authors due to valuable application in industry. A. A. Atai and D. Lak [1] presented analytic investigation of effect of electric field on elasto-plastic response of a functionally graded piezoelectric hollow sphere. M. Jabbari and Aghdam [2] investigated asymmetric thermal stresses of hollow FGM cylinders with piezoelectric internal and external layers. M. Jabbari and A. R. Barati [3] presented analytical solution for the thermopiezoelastic behavior of a smart functionally graded material hollow sphere under radially symmetric loadings. A. Atrian et al. [4] presented thermo-electromechanical behavior of functionally graded piezoelectric hollow cylinder under non-axisymmetric loads. A. R. Barati and M. Jabbari [5] analyzed two-dimensional thermopiezoelastic of a smart FGM hollow sphere. Nguyen Dinh Duc et al. [6] presented nonlinear dynamic analysis and vibration of shear deformable piezoelectric FGM double curved shallow shells under damping-thermo-electro-mechanical loads. Xue-Qian Fang et al. [7] presented size-dependent effects on electromechanical response of multilayer piezoelectric nano-cylinder under electro-elastic waves. M. Saadatfar and A. Rastgoo [8] presented an analytic solution to the axisymmetric problem of a radially polarized radially orthotropic piezoelectric hollow cylinder with thermal gradient. Ghorbanpour Arani et al. [9] presented electro-thermo-mechanical behaviors of FGM spheres using closed form solution and FEM methods. Ibrahim A. Abbas [10] studied the electro–magneto–thermo-elastic analysis problem of an infinite FGM hollow cylinder based upon LS theory. M. Sabzikar Boroujerdy et al. [11] investigated axisymmetric snap-through behavior of piezo FGM shallow clamped spherical shells under thermo-electro-mechanical loading. Wang and Xu [12] presented the effect of material inhomogeneity on electromechanical behaviors of functionally graded piezoelectric spherical structures. Chih Ping Wu [13] presented unified formulation of PVD-based finite cylindrical layer methods for functionally graded material sandwich cylinders. Farshad Ehsani and Farzad Ehsani [14] analyzed the one dimensional non steady state temperature distribution in a hollow thick cylinder of FGM with non-uniform heat generation by homotopy perturbation method. Dai et al. [15] presented analytical solutions

* Corresponding author.
E-mail address: davoodlak@gmail.com (D. Lak).

http://dx.doi.org/10.1016/j.ijpvp.2017.06.007
0308-0161/

of stresses in FG piezoelectric hollow Structures. Jafari Fesharaki et al. [16] studied two-dimensional Solution for electro-mechanical behavior of FG piezoelectric hollow cylinder. Alashti et al. [17] presented three-dimensional thermo-elastic analysis of a FG cylindrical shell with piezoelectric layers by DQ method. Ghorbanpour Arani et al. [18] analyzed electro-thermo-mechanical creep and time-dependent behavior of FGM spheres. Ghorbanpour Arani et al. [19] presented the axial and torsional wave propagation in a double-walled carbon nanotube embedded on elastic foundations using nonlocal continuum shell theory. Mojtaba Sadeghian and Hamid Ekhteraei Toussi [20] presented solutions to the problem of thermomechanical yielding in FGM cylindrical vessels. M. Saadatfar, A.S. Razavi [21] studied stress distribution in a piezoelectric hollow cylinder with thermal gradient.

The aim of this paper is to investigate the elasto-plastic response of a functionally graded material cylinder vessel subjected to electro-thermo-mechanical loads. Using hook's low, strain-displacement relations and equilibrium equation, the governing equation obtains as known Navier equation. By solution of Navier equation, radial displacement obtains and all other unknown parameters such as radial and tangential stresses and strains, potential and electric intensity can be obtained. The FG material properties such as potential and electric intensity coefficients, elastic constants and thermal expansion and conductivity are considered to be a power law function of radius. The vessel is in steady-state condition and the behavior of material is assumed elastic perfectly plastic. The constituent in inner radius is chosen PZT-4 as a known piezoelectric material. The experimental yield stress and elasto-plastic behavior of PZT-4 studied in Ref. [23] is used in this paper.

2. Analysis of the problem

Fig. 1 shows an inhomogeneous piezoelectric hollow cylinder with inner radius R_{in} and outer radius R_{out}, subject to pressure, temperature, and electric potential on the inner and outer surface designated respectively by P_{in}, T_{in}, φ_{in} , and P_{out}, T_{out}, and φ_{out}. The inhomogeneous thermo-electro-mechanical properties vary in the radial direction by power law as:

$$X = X_{\text{in}}\left(\frac{r}{R_{\text{in}}}\right)^{n} \quad (1)$$

in which X is the general property of the cylinder such as the elastic, thermal expansion, piezoelectric and dielectric coefficients at radius r, and X_{in} is the value of the same property at the inner surface, n is the inhomogeneity exponent with $n = 0$ corresponding to the homogeneous case. Introducing the reference parameter X_0:

$$X_0 = \frac{X_{\text{in}}}{R_{\text{in}}^{n}} \quad (2)$$

equation (1) can be rewritten as:

$$X = X_0 r^{n} \quad (3)$$

In the equations that follow, a zero-indexed parameter is considered to be referential based on definition (2).

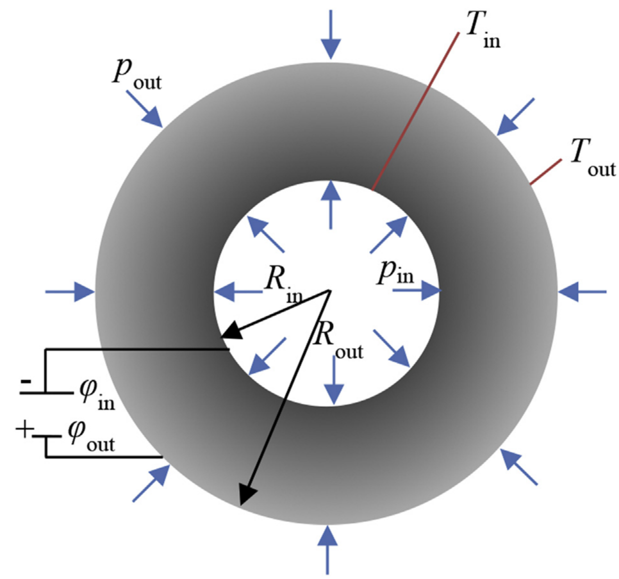

Fig. 1. Radially graded cylindrical vessel.

2.1. Elastic analysis

The thermo-electro-mechanical constitutive law in the cylindrical coordinates (r, θ, z) is stated as:

$$C_{12}\varepsilon_\theta - e_r E_r - (C_{11}\alpha_r + C_{12}\alpha_\theta)T + C_{11}\varepsilon_r = \sigma_r \quad (4)$$

$$C_{22}\varepsilon_\theta - e_\theta E_r - (C_{12}\alpha_r + C_{22}\alpha_\theta)T + C_{12}\varepsilon_r = \sigma_\theta \quad (5)$$

$$\varepsilon_r E_r - (e_r\alpha_r + e_\theta\alpha_\theta)T + e_\theta\varepsilon_\theta + e_r\varepsilon_r = D_r \quad (6)$$

where $C_{ij} = C_{ji}$, ε_i, σ_i, e_i, ε_i α_i, $(i = r, \theta,$ or $1, 2)$, are the elastic constants, strain, stress, piezoelectric coefficient, dielectric permittivity, and thermal expansion coefficient, respectively, T is temperature, $E_r = -d\varphi/dr$ is the electric field strength (φ is the electric potential), and D_r is the flux density. The distribution of temperature along the radial direction is given by Ref. [22]:

$$T = C_1 r^{-n} + C_2 \quad (7)$$

in which constants C_1 and C_2 can be obtained using the thermal boundary conditions. Based on problem symmetry, the strain-displacement relations are:

$$\varepsilon_r = \frac{du}{dr} \quad (8)$$

$$\varepsilon_\theta = \frac{u}{r} \quad (9)$$

where u is the radial displacement. Neglecting the body forces and free charge density, the equilibrium equation and electrostatic equation for displacement current in the FGP hollow cylinder are given by: